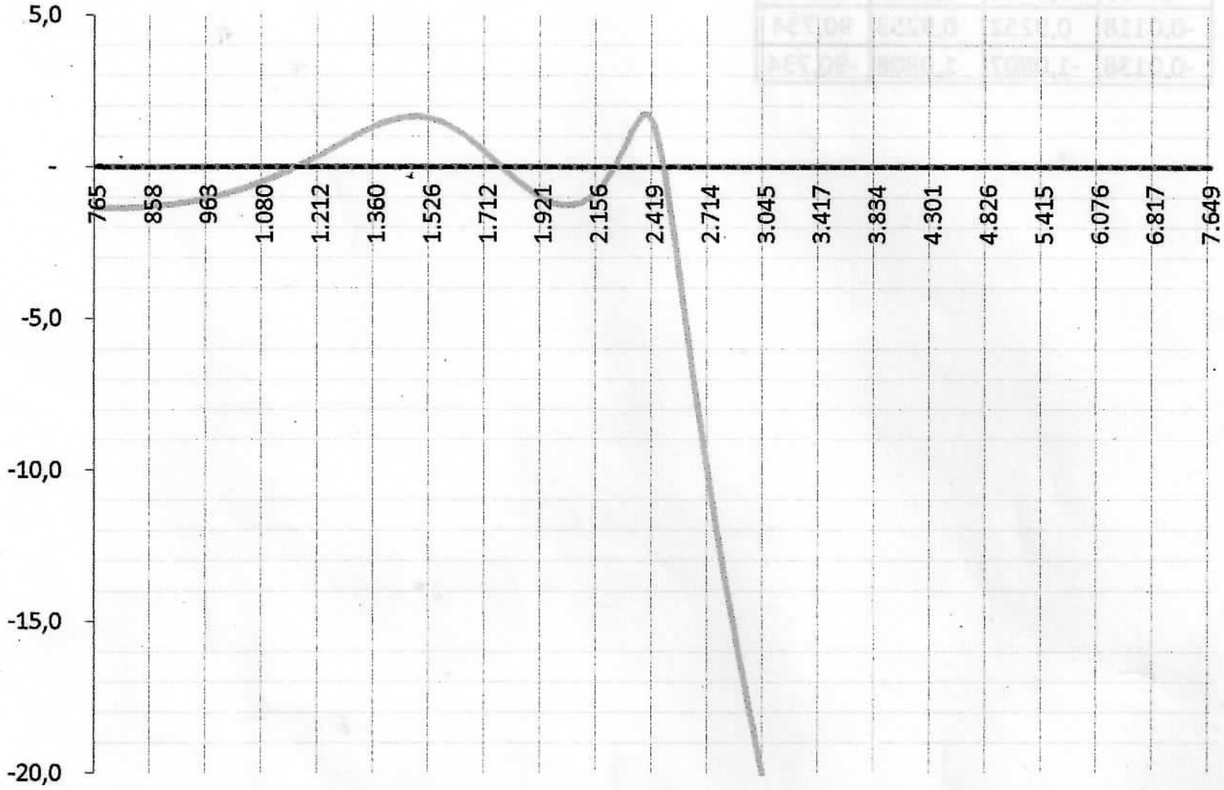
Versão 3

# Filtros Ativos
# Roteiro para projeto.

Prof. Claudio Barbalho





Bibliografia:

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Sallen-key
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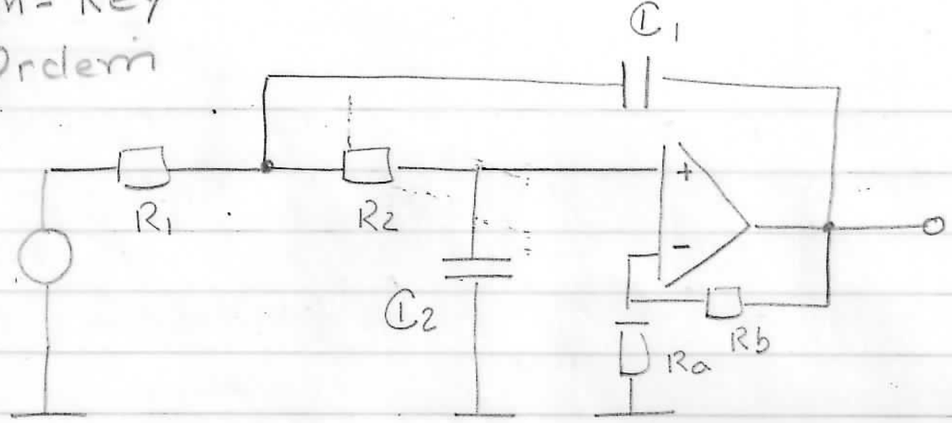



Passa Baixa
Sallen-key

$$\frac{E_2}{E_1} = \frac{\dfrac{\mu}{R_1 R_2 C_1 C_2}}{s^2 + \left(\dfrac{1}{R_1 C_1} + \dfrac{1}{R_2 C_1} + \dfrac{1-\mu}{R_2 C_2}\right) s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

onde $\mu = 1 + \dfrac{R_b}{R_a}$

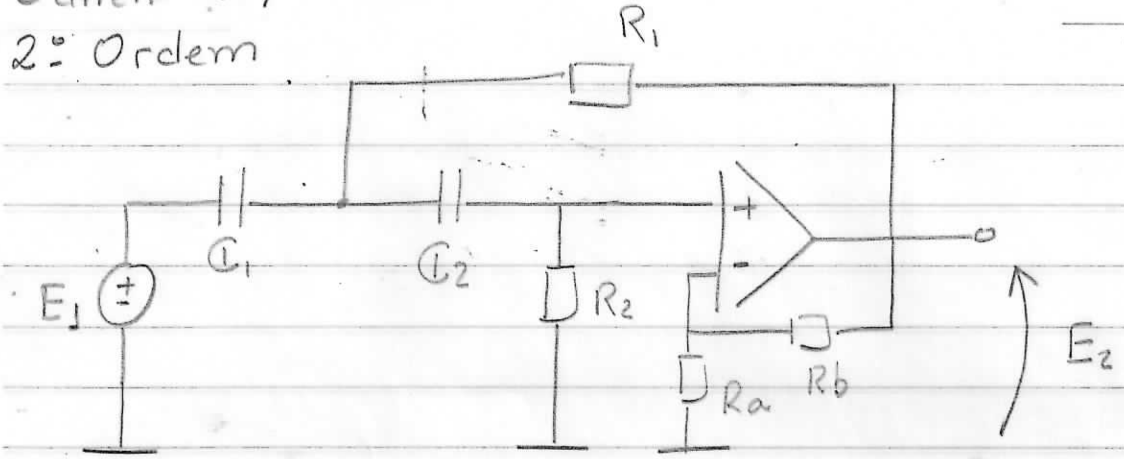
$$H(s) = \frac{G \omega_0^2}{s^2 + \left(\dfrac{\omega_0}{Q}\right) s + \omega_0^2}$$

onde $\omega_0 = \dfrac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

$$Q = \frac{\omega_0}{\dfrac{1}{R_1 C_1} + \dfrac{1}{R_2 C_1} + \dfrac{1-\mu}{R_2 C_2}}$$

$$G = \mu$$

Passa Alta
Sallen - Key
2º Ordem




$$\frac{E_2}{E_1} = \frac{\mu s^2}{s^2 + \left( \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} + \frac{1-\mu}{R_1 C_1} \right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$H_{HP}(s) = \frac{G s^2}{s^2 + \left( \frac{\omega_0}{Q} \right) s + \omega_0^2}$$

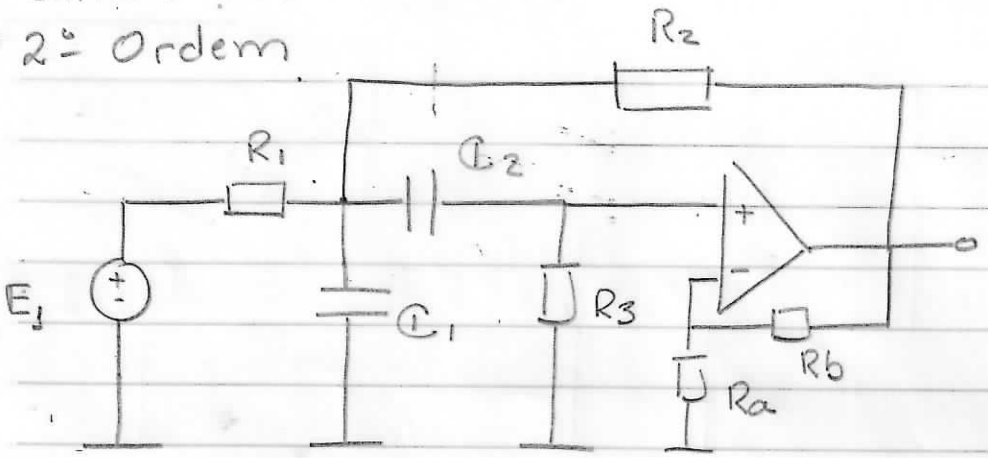
onde $\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

$$Q = \frac{\omega_0}{\frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} + \frac{1-\mu}{R_1 C_1}}$$

$$G = \mu$$

$$\mu = 1 + \frac{R_b}{R_a}$$

· Passa Faixa
Sallen-Key
2ª Ordem




$$\frac{E_2}{E_J} = \frac{\dfrac{\mu}{R_1 C_1}\, s}{s^2 + \left(\dfrac{1}{R_1 C_1} + \dfrac{1}{R_3 C_2} + \dfrac{1}{R_3 C_1} + \dfrac{1-\mu}{R_2 C_1}\right) s + \dfrac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

$$H_{BP}(s) = \frac{G\left(\dfrac{\omega_0}{Q}\right) s}{s^2 + \left(\dfrac{\omega_0}{Q}\right) s + \omega_0^2}$$

onde $\omega_0 = \sqrt{\dfrac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$

$$Q = \frac{\omega_0}{\dfrac{1}{R_1 C_1} + \dfrac{1}{R_3 C_2} + \dfrac{1}{R_3 C_1} + \dfrac{1-\mu}{R_2 C_1}}$$

$$G = \frac{\dfrac{\mu}{R_1 C_1}}{\dfrac{1}{R_1 C_1} + \dfrac{1}{R_3 C_2} + \dfrac{1}{R_3 C_1} + \dfrac{1-\mu}{R_2 C_1}}$$

$$\mu = 1 + \frac{R_b}{R_a}$$

# Projeto de Filtros Ativos de 2ª Ordem

(sem o uso dos polinômios normalizados de Butterworth e Chebyshev)

Projeto sem controle do ganho

## Roteiro para Projeto: Passa Baixa

Fazendo $C_1 = C_2 = 1$ e $R_1 = R_2 = R$ temos:

$$\omega_0 = \frac{1}{R} \longrightarrow R = \frac{1}{\omega_0}$$

$$Q = \frac{1}{3-\mu} \longrightarrow \mu = 1 + \frac{R_b}{R_a} = 3 - \frac{1}{Q} \quad \therefore \frac{R_b}{R_a} = 2 - \frac{1}{Q}$$

$$G = \mu .$$

Exemplo:
Projete um filtro P. Baixa de 2ª Ordem com $f_o = 500\ Hz$ e $Q = 0{,}7$.

$C_1 = C_2 = 1$ $\quad R = \frac{1}{\omega_0} = 1$ p/ $\omega_0 = 1\ r/s$

$R_1 = R_2 = R$

$$\frac{R_b}{R_a} = 2 - \frac{1}{0{,}7} = \frac{10\ k\Omega}{17{,}5\ k\Omega}$$

Deslocamento de frequência e de impedâncias:

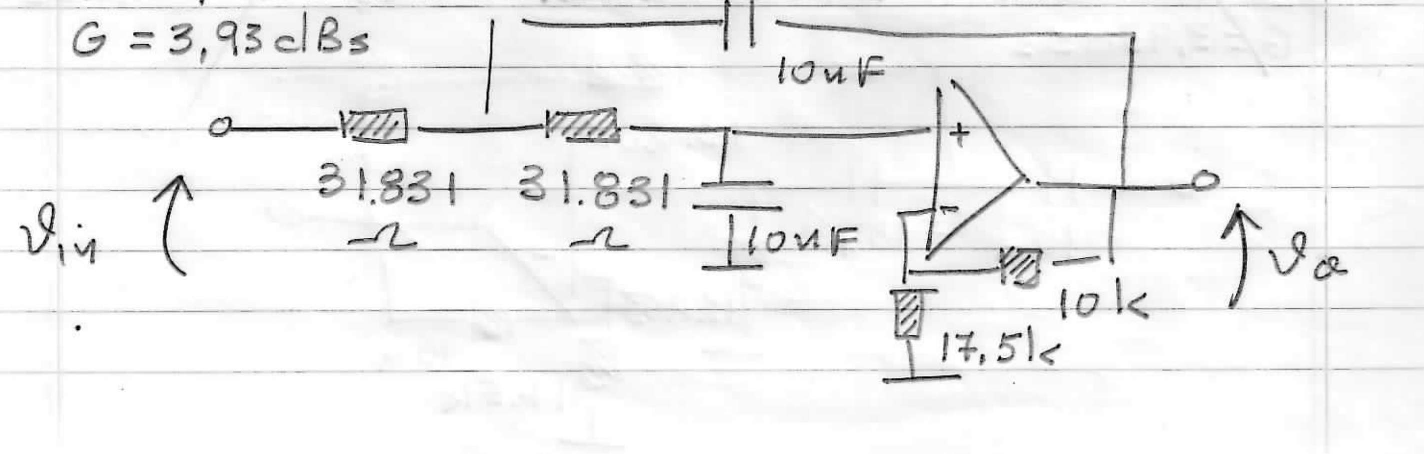
$$\omega_0 = 1 \xrightarrow{\times k_f} \omega_0 = 2\pi 500$$

$$C_s = 1 \xrightarrow{\times \frac{1}{k_f}} \frac{1}{k_f} \xrightarrow{\times \frac{1}{k_z}} 10\ nF$$

$$R_s = 1 \xrightarrow{\times k_z} 31.831\ \Omega$$

$G = 1{,}57$

$G = 3{,}93\ dBs$



Roteiro para Projeto: Passa Alta
Da mesma forma que para P. Baixa

Fazemos $C_1 = C_2 = 1$ e $R_1 = R_2 = R$ e temos:

$$\omega_0 = \frac{1}{R} \longrightarrow R = \frac{1}{\omega_0} \longrightarrow R\Big|_{\omega_0 = 1} = 1$$

$$Q = \frac{1}{3-\mu} \longrightarrow \mu = 3 - \frac{1}{Q} = 1 + \frac{R_b}{R_a} \longrightarrow \frac{R_b}{R_a} = 2 - \frac{1}{Q}$$

$G = \mu$.

Exemplo: Projete um P. Alta de 2ª Ordem com $f_0 = 1.200\ Hz$ e $Q = 0,7$.

$C_1 = C_2 = 1$
$R_1 = R_2 = R$

$$\longrightarrow R\Big|_{\omega_0=1} = 1$$

$$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = \frac{10}{17,5}$$

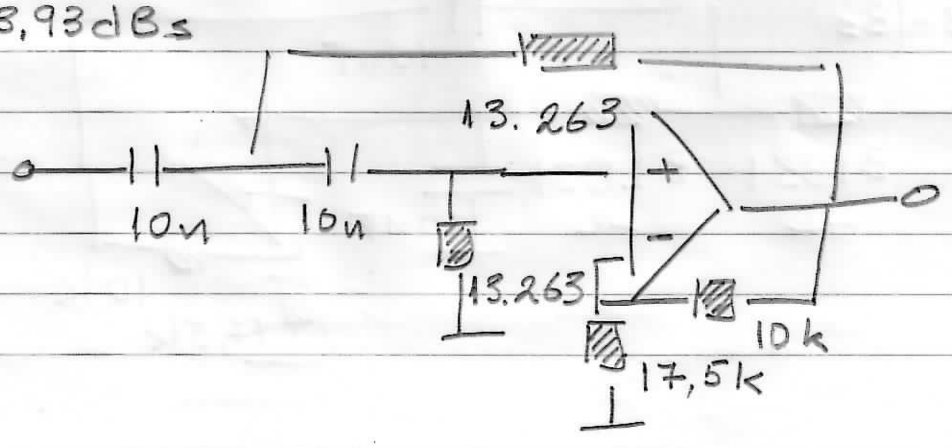
Deslocamentos de Frequências e de impedâncias:

$\omega_0 = 1 \xrightarrow{\times k_f} 2\pi 1.200$

$C_s = 1 \xrightarrow{\times \frac{1}{k_f}} \xrightarrow{\times \frac{1}{k_z}} 10n$

$G = 1,57$ $\qquad R_s = 1 \xrightarrow{\times k_z} 13.263$

$G = 3,93\ dBs$



# Roteiro para Projeto — Passa Faixa

Fazemos $C_1 = C_2 = 1$ e $R_1 = R_2 = R_3 = R$ e temos:

$$\omega_0 = \frac{\sqrt{2}}{R} \rightarrow R = \frac{\sqrt{2}}{\omega_0} \rightarrow R\Big|_{\omega_0 = 1} = \sqrt{2}$$

$$Q = \frac{\sqrt{2}}{4-\mu} \rightarrow \mu = 1 + \frac{R_b}{R_a} = 4 - \frac{\sqrt{2}}{Q} \rightarrow \frac{R_b}{R_a} = 3 - \frac{\sqrt{2}}{Q}$$

$$G = \frac{\mu}{4-\mu}$$

Exemplo: Projete um Filtro Passa Faixa de 2ª Ordem com $f_{c_1} = 1.500\ Hz$ e $f_{c_2} = 1.800\ Hz$.

$$f_0 = \sqrt{f_{c1} \cdot f_{c2}} = 1.643,17$$
$$B = f_{c2} - f_{c1} = 300$$
$$Q = f_0 / B = 5,477$$

$C_1 = C_2 = 1$
$R_1 = R_2 = R_3 = R$

$$\rightarrow R = \frac{\sqrt{2}}{\omega_0} \rightarrow R\Big|_{\omega_0 = 1} = \sqrt{2}$$

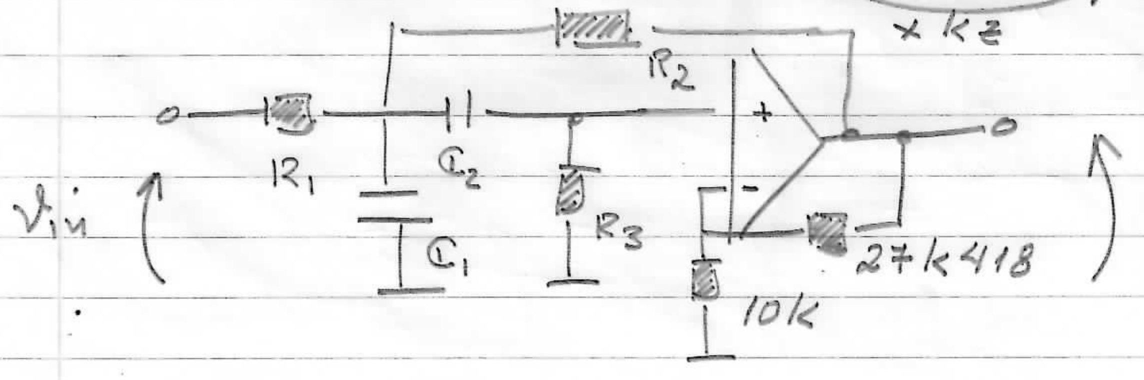
$$\frac{R_b}{R_a} = 3 - \frac{\sqrt{2}}{Q} = \frac{27,418\ k}{10,00\ k}$$

Deslocamentos:

$\omega_0 = 1 \xrightarrow{\times k_f} 2\pi\, 1.643$

$C_s = 1 \xrightarrow{\times \frac{1}{k_f}} \xrightarrow{\times \frac{1}{k_z}} 10\ nF$

$R = \sqrt{2} \xrightarrow{\times k_z} 13.699$



Projeto com
controle
do
ganho

# Passa Baixa com controle de G.

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad G = \mu = 1 + \frac{R_b}{R_a}$$

$$Q = \frac{\omega_0}{\frac{1}{R_1 C_1} + \underbrace{\frac{1}{R_2 C_1} + \frac{1-\mu}{R_2 C_2}}_{=0}}$$

$$\frac{1}{\cancel{R_2} C_1} + \frac{1-\mu}{\cancel{R_2} C_2} = 0$$

$$\frac{C_2}{C_1} = \mu - 1$$

$$C_2 = (\mu - 1) C_1$$

$$= \frac{\frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}{\frac{1}{R_1 C_1}} = \sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

$$(Q)^2 = \left(\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right)^2$$

$$Q^2 = \frac{R_1 C_1}{R_2 C_2} = \frac{R_1}{R_2(\mu - 1)}$$

$$R_1 = Q^2 (\mu - 1) R_2$$

Então temos:

Definidos $G$, $Q$, $f_0$, $C_1$ e $R_2$ calculamos:

1. $\frac{R_b}{R_a} = G - 1$
2. $C_2 = (\mu - 1) C_1$
3. $R_1 = Q^2 (\mu - 1) R_2$
4. $\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \neq 1$ !!

Exemplo:

Projete um P.Baixa com os seguintes caraquiterísticas:

2ª Ordem

$f_o = 1.000$

$G = 11$

$Q = 0,7$

Fazemos $C_1 = 1$ e $R_2 = 1$

$$\frac{Rb}{Ra} = G - 1 = 10 = \frac{100k}{10k}$$

$$C_2 = (\mu - 1)\, C_1 = 10$$

$$R_1 = Q^2 (\mu - 1) R_2 = 4,9$$

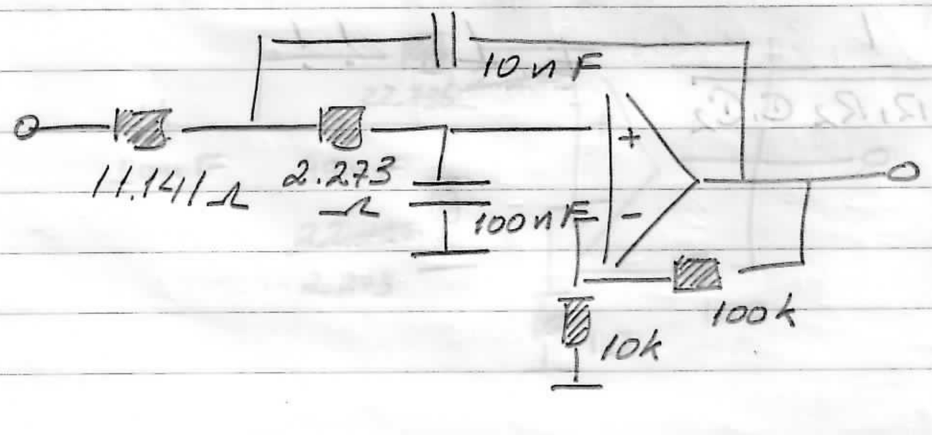
$$\omega_o = \frac{1}{\sqrt{1 \times 10 \times 1 \times 4,9}} = \frac{1}{7} = 0,1428$$

Deslocamentos:

$$\omega_o = 0,1428 \xrightarrow{\times k_f} 2\pi 1.000$$

$$\begin{matrix} c = 1 \\ c = 10 \end{matrix} \xrightarrow{\times \frac{1}{k_f}} \xrightarrow{\times \frac{1}{k_z}} \begin{matrix} 10n \\ 100n \end{matrix}$$

$$\begin{matrix} R_1 = 4,9 \\ R_2 = 1 \end{matrix} \xrightarrow{\times k_z} \begin{matrix} 11.141 \\ 2.273 \end{matrix}$$



# Passo Alta com controle de G

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad G = \mu = 1 + \frac{R_b}{R_a}$$

$$Q = \frac{\omega_0}{\frac{1}{R_2 C_2} + \underbrace{\frac{1}{R_2 C_1} + \frac{1-\mu}{R_1 C_1}}_{=0}} = \frac{\frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}{\frac{1}{R_2 C_2}} = \sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

$$\frac{1}{R_2 \cancel{C_1}} + \frac{1-\mu}{R_1 \cancel{C_1}} = 0 \qquad (Q)^2 = \left(\sqrt{\frac{R_2 C_2}{R_1 C_1}}\right)^2$$

$$\frac{R_1}{R_2} = \mu - 1 \qquad Q^2 = \frac{R_2 C_2}{R_1 C_1}$$

$$R_1 = (\mu - 1) R_2 \qquad C_2 = Q^2 (\mu - 1) C_1$$

Resumindo temos:

$$! \; \frac{R_b}{R_a} = G - 1$$

$$C_2 = Q^2 (\mu - 1) C_1$$

$$R_1 = (\mu - 1) R_2$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \neq 1 \quad !!!$$

Exemplo:
Projete um Filtro P. Alta com os seguintes caracteristicas:

2ª Ordem
f = 1.000
G = 11
Q = 0,7

Fazemos $C_1 = 1$ e $R_2 = 1$

$$\frac{R_b}{R_a} = 11 - 1 = 10 = \frac{100k}{10k}$$

$$C_2 = Q^2(\mu - 1)\,C_1 = 4{,}9$$

$$R_1 = (\mu - 1)\,R_2 = 10$$

$$\omega_0 = \frac{1}{\sqrt{1 \times 1 \times 4{,}9 \times 10}} = \frac{1}{7} = 0{,}1428$$

Deslocamentos:

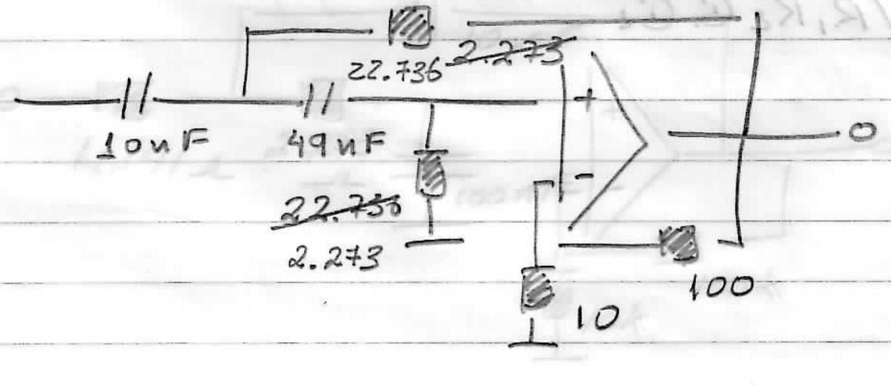
$$\omega_0 = 0{,}1428 \xrightarrow{\times k_f} 2\pi\, 1.000$$

$$C = 1 \xrightarrow{\times \frac{1}{k_f}} \xrightarrow{\times \frac{1}{k_f}} 10nF$$

$$C = 4{,}9 \qquad 49nF$$

$$R_2 = 1 \longrightarrow 2.273$$

$$R_1 = 10 \longrightarrow 22.736$$



# Passa Faixa com controle de G.

$$\omega_0 = \sqrt{\frac{R_1+R_2}{C_1 C_2 R_1 R_2 R_3}} \qquad \mu = 1 + \frac{R_b}{R_a}$$

$$Q = \frac{\omega_0}{\frac{1}{R_1 C_1} + \underbrace{\frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + \frac{1-\mu}{R_2 C_1}}_{\simeq 0}} = \frac{\sqrt{\frac{R_1+R_2}{C_1 C_2 R_1 R_2 R_3}}}{\frac{1}{R_1 C_1}} = \sqrt{\frac{(R_1+R_2) R_1 C_1}{C_2 R_2 R_3}}$$

$$G = \frac{\frac{\mu}{R_1 C_1}}{\frac{1}{R_1 C_1} + \underbrace{\frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + \frac{1-\mu}{R_2 C_1}}_{\simeq 0}} \rightarrow G = \mu$$

$$e \; C_2 = a C_1$$

$$\frac{1}{R_3 a \cancel{C_1}} + \frac{1}{R_3 \cancel{C_1}} + \frac{1-\mu}{R_2 \cancel{C_1}} = 0$$

$$R_3 = \left(\frac{a+1}{a}\right) \times \left(\frac{1}{\mu-1}\right) \times R_2$$

$$R_1 = \frac{-R_2 \pm \sqrt{R_2^2 + 4aQ^2 R_2 R_3}}{2}$$

Resumindo temos:

$$a = \frac{C_2}{C_1}$$

$$R_3 = \left(\frac{a+1}{a}\right)\left(\frac{1}{\mu-1}\right) R_2 \qquad \mu = 1 + \frac{R_b}{R_a} = G$$

$$\frac{R_b}{R_a} = G - 1$$

$$R_1 = \frac{-R_2 \pm \sqrt{R_2^2 + 4aQ^2 R_2 R_3}}{2}$$

$$\omega_0 = \sqrt{\frac{R_1+R_2}{C_1 C_2 R_1 R_2 R_3}} \neq 0!!$$

Exemplo:
Projete um filtro P. Faixa com as seguintes caracteristicas:

$G = 10$ | $f_0 = \sqrt{f_{c1} f_{c2}} = 1.643$
$f_{c1} = 1.500\ Hz$ | $B = f_{c2} - f_{c1} = 300$
$f_{c2} = 1.800\ Hz$ | $Q = f_0 / B = 5,477$

Fazemos $a = 0,22 = \dfrac{C_2}{C_1} \to \dfrac{2,2}{10}$

e $R_2 = 1$ como "$\mu = G$" teremos:
nestas condições

$$R_3 = \left(\frac{1+a}{a}\right)\left(\frac{1}{\mu - 1}\right) R_2 = 0,616161$$

$$R_1 = \frac{-1 + \sqrt{1 + 4 \times 0,22 \times (5,477)^2 \times 1 \times 0,616161}}{2}$$

$$R_1 = 1,5776$$

$$w_0 = \sqrt{\frac{1,5776 + 1}{2,2 \times 10 \times 1,5776 \times 1 \times 0,616161}} = 0,34717$$

Deslocamentos $\qquad \dfrac{R_b}{R_a} = G - 1 = 9$

$w_0' = 0,34717 \xrightarrow{\times k_f} 2\pi 1.643$

$C = 10 \xrightarrow{\times 1/k_f} 10n$

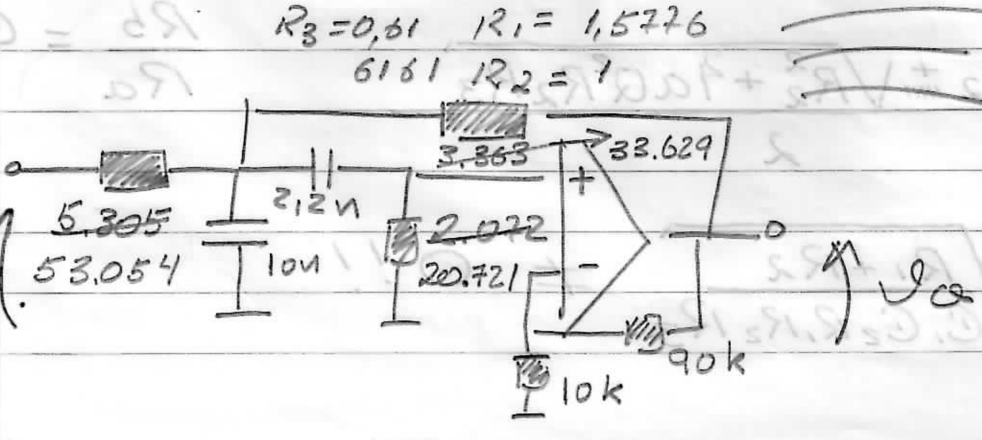
$C = 2,2 \to 2,2n$

$R_3 = 0,616161 \quad R_1 = 1,5776 \quad R_2 = 1$

$R_2 = \cancel{3.363}$ 33.629

$R_1 = \cancel{5.305}$ 53.054

$R_3 = \cancel{2.072}$ 20.721



# Projeto de Filtros Ativos

(com o uso dos polinômios normalizados de Butterworth e Chebyshev)

# Filtros Ativos

Com a utilização dos polinômios de Butterword ou Chebyshev e dos circuitos de Sallen-Key.

1. P. Baixa → utiliza-se diretamente os coeficientes dos polinômios:

$$\frac{1}{(s^2 + bs + c)} \rightarrow \begin{aligned} \omega_o^2 &= c \\ \frac{\omega_o}{Q} &= b \end{aligned}$$

2. P. Alta → faz-se $s = \frac{1}{s}$ e teremos

$$\frac{1}{\frac{1}{s^2} + \frac{b}{s} + c} = \frac{s^2/c}{\left(s^2 + \frac{b}{c}s + \frac{1}{c}\right)}$$

$$\left.\begin{aligned} \omega_o^2 &= 1/c \\ \frac{\omega_o}{Q} &= \frac{b}{c} \end{aligned}\right\}$$ * ver pag. 40

3. P. Faixa → faz-se $s = \frac{s^2 + \omega_o^2}{Bs}$

$$\frac{1}{(s^2 + bs + c)} = \frac{1}{(s - s_1)(s - s_2)} =$$

$$\frac{1}{\left(\frac{s^2 + \omega_o^2}{Bs} - s_1\right)\left(\frac{s^2 + \omega_o^2}{Bs} - s_2\right)}$$

→

e teremos:

$$\underbrace{\left(\frac{BS}{S^2 - s_1 B S + \omega_0^2}\right)}_{A}\underbrace{\left(\frac{BS}{S^2 - s_2 B + \omega_0^2}\right)}_{B}$$

onde $s_{1,2}$ são complexos conjugados. Resolvendo agora A e B teremos:

$$S^2 - s_1 B S + \omega_0^2 \longrightarrow s_3, \; s_4$$

$$S^2 - s_2 B S + \omega_0^2 \longrightarrow s_5, \; s_6$$

,

onde $s_3$ é o complexo conjugado de $s_5$ e $s_4$ de $s_6$ (ver ~~último~~ exercício resolvido), e teremos então: → pag 30 a 37

$$\frac{BS}{\underbrace{(S - s_3)}_{1}\underbrace{(S - s_5)}_{2}} \times \frac{BS}{\underbrace{(S - s_4)}_{3}\underbrace{(S - s_6)}_{4}}$$

multiplicando (1) e (2) e (3) e (4) teremos como resultado polinômios de 2º grau com a, b e c reais: $\frac{BS}{S^2 + b_1 S + C_1} \times \frac{BS}{S^2 + b_2 S + C_2}$, que são facilmente realizaveis pelos circuitos de Sallen-Key.

Projetos sem
o controle
do
Ganho

Exemplos:

1. Passa Baixa $f_c = 200\ Hz$ 4º Ordem.
1.1 Butterworth

$$\underbrace{(s^2 + 0{,}765367\,s + 1)}_{A}\ \underbrace{(s^2 + 1.847759\,s + 1)}_{B}$$

A) $\omega_0 = 1$ , $\dfrac{\omega_0}{Q} = 0{,}765367 = \dfrac{1}{Q}$

$\left.\begin{array}{l} C_1 = C_2 = 1F \\ R_1 = R_2 = R \end{array}\right\}$ temos que $\omega_0 = \dfrac{1}{RC} \rightarrow$

$\rightarrow R = 1\ \Omega$ , e que $\mu = 3 - \dfrac{1}{Q} = 2{,}235$

$\mu = 1 + \dfrac{R_b}{R_a}$ , $\dfrac{R_b}{R_a} = 1{,}235$ , $R_b = 1{,}235\,R_a$

$\Longrightarrow R\cancel{b}_a = 10K \longrightarrow R\cancel{a}_b = 12k350\ \Omega$ *

$f_c = 200\ Hz \longrightarrow \omega_c = 1.256\ r/s$

Todos os polinômios estão normalizados p/ $1\ r/s$ então:

$$\begin{array}{l} \omega_c = 1 \\ C = 1 \end{array} \xrightarrow[1{,}256]{K_f} \begin{array}{l} \omega_c = 1{,}256 \\ C = 795\ \mu F \end{array}$$

$$\begin{array}{l} C = 795\ \mu F \\ R = 1 \end{array} \xrightarrow[7{,}957]{k_z} \begin{array}{l} C = 100\ n \ * \\ R = 7{,}957\ \Omega \ * \end{array}$$

B) $\omega_0 = 1$, $\frac{\omega_0}{Q} = 1{,}847759 = \frac{1}{Q}$

$\omega_0 = \frac{1}{RC} \rightarrow \omega_0 = 1$, $C = 1F$, $R = 1\Omega$

$\mu = 3 - \frac{1}{Q} = 1{,}1522 = 1 + \frac{Rb}{Ra}$

$\frac{Rb}{Ra} = 0{,}1522 \rightarrow Ra = 6{,}568\ Rb$

$Ra = 65.680$, $Rb = 10k\ \Omega$ *

$f_c = 200\ Hz \rightarrow \omega_c = 1.257\ r/s$

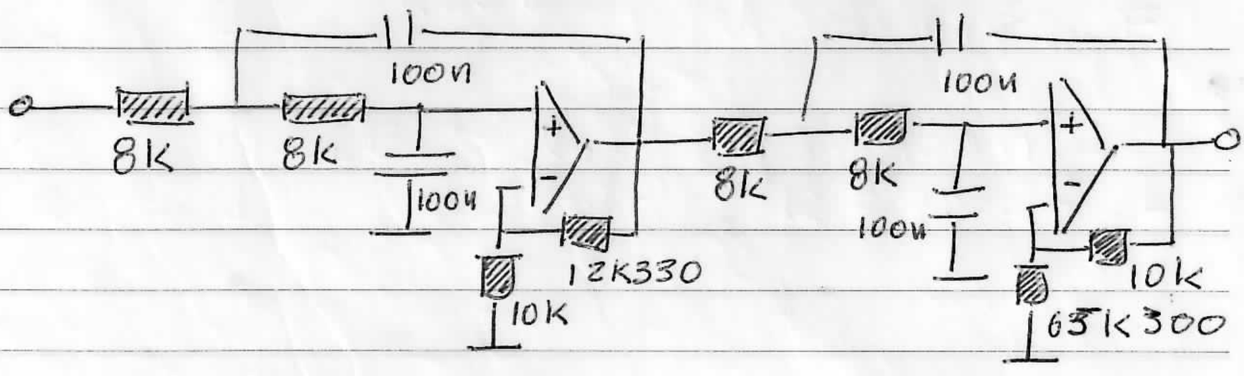
$\omega_0 = 1 \xrightarrow[1.257]{k_f} \omega_0 = 1.257$

$C = 1 \longrightarrow C = 795\ \mu F$

$C = 795\ \mu F \xrightarrow[7.955]{k_z} C = 100\ nF$ *

$R = 1 \longrightarrow R = 7.955\ \Omega$ *

| | Estagio 1 | Estagio 2 |
|---|---|---|
| $R =$ | $4k7 + 3k3\ \Omega$ | $4k7 + 3k3\ \Omega$ |
| $C =$ | $100n\ F$ | $100n\ F$ |
| $Ra =$ | $10k\ \Omega$ | $62k + 3k3\ \Omega$ |
| $Rb =$ | $12k + 330\ \Omega$ | $10k\ \Omega$ |



# 1.2 Chebyshev

$(s^2 + 0{,}209775\,s + 0{,}928675) \times \longrightarrow A$

$(s^2 + 0{,}506440\,s + 0{,}221568) \longrightarrow B$

A) $\omega_0 = \dfrac{1}{RC} = \sqrt{0{,}928675} = 0{,}9637$

$\dfrac{\omega_0}{Q} = 0{,}209775 \longrightarrow Q = 4{,}5938$

$\dfrac{1}{RC} = 0{,}9637 \longrightarrow C = 1F \;,\; R = 1{,}038$

$\mu = 3 - \dfrac{1}{Q} = 1 + \dfrac{R_b}{R_a} = 2{,}782$

$\dfrac{R_b}{R_a} = 1{,}782 \longrightarrow R_a = 10K$ *

$R_b = 17{,}82\,K$ *

Polinomio normalizado p/ $\omega_c = 1$

Deslocando $\omega_c$ para $2\pi f = 1.256{,}64$

temos $k_f = 1.256{,}64 \Rightarrow$

$\omega_c = 1 \xrightarrow{k_f} \omega_c = 1.256{,}64$

$C = 1 \xrightarrow{1.256{,}4} C = 795{,}8\,\mu F$

$C = 795{,}8\mu \xrightarrow{k_z \; 7.957} C = 100\,nF$ *

$R = 1{,}038 \longrightarrow R = 8.260\,\Omega$ *

B) $\omega_0 = \sqrt{0{,}221568} = 0{,}47071 = \dfrac{1}{RC}$

$C = 1F \;,\; R = 2{,}12441\,\Omega$

$\dfrac{\omega_0}{Q} = 0{,}506440 \longrightarrow Q = 0{,}92944$

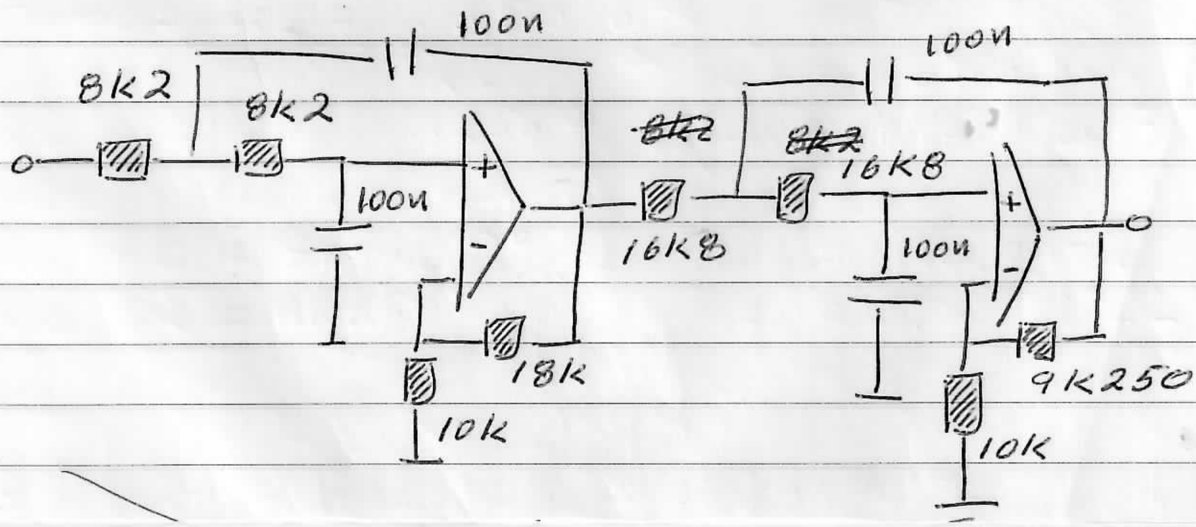
$$\mu = 3 - \frac{1}{Q} = 1.9241 = 1 + \frac{Rb}{Ra}$$

$$\frac{Rb}{Ra} = 0{,}9241 \rightarrow \begin{array}{l} Ra = 10k\ \Omega \quad * \\ Rb = 9.241\ \Omega \quad * \end{array}$$

$$\begin{array}{l} \omega_c = 1 \\ C = 1 \end{array} \xrightarrow{kf} \begin{array}{l} \omega_o = 1.256{,}63 \\ C = 795{,}8\ \mu F \end{array}$$

$$\begin{array}{l} C = 795{,}8\mu F \\ R = 2{,}1244 \end{array} \xrightarrow[7.958]{kz} \begin{array}{l} 100nF \quad * \\ R = 16.905\ \Omega \quad * \end{array}$$

| | Estagio 1 | Estagio 2 |
|---|---|---|
| R = | 8.260 Ω | 16.905 Ω |
| C = | 100 nF | 100n F |
| Ra = | 10K Ω | 10k Ω |
| Rb = | 17K820 Ω | 9.241 Ω |



3. Filtro Passa Faixa : $f_{c1} = 900\,Hz$
( 2ª Ordem ) $f_{c2} = 1.100\,Hz$

$B = f_{c2} - f_{c1} = 200$

$f_0 = \sqrt{f_{c1} \times f_{c2}} = 995\,Hz$

$$Q = \frac{f_0}{B} = \frac{995}{200} = 4{,}9749$$

p/ $\omega_0 = 1$ , $B = \frac{\omega_0}{Q} = 0{,}201$

3.1 Butterworth 2ª Ordem

$$\frac{1}{(s+1)} \rightarrow \frac{1}{(s - s_k)} \; , \; s_k = -1$$

substituindo $s$ por $\frac{s^2+\omega_0^2}{Bs}$

teremos $$\frac{1}{\left(\frac{s^2+\omega_0^2}{Bs} - s_k\right)} \rightarrow$$

$$\rightarrow \frac{Bs}{s^2 - s_k B s + \omega_0^2}$$ que para

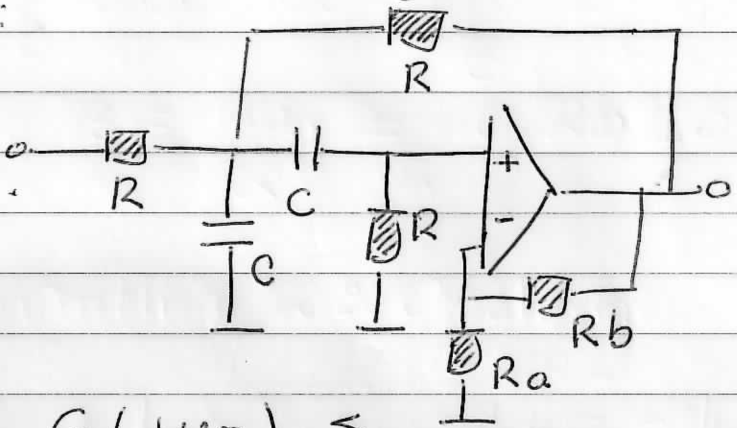
$s_k = -1$, $B = 0{,}201$ e $\omega_0 = 1$ temos:

$$\frac{0{,}201\,s}{(s^2 + 0{,}201\,s + 1)} \longrightarrow$$

$(s^2 + 0{,}201 s + 1)$ é o polinômio de Butterworth p/ filtro Passa Faixa de 2ª Ordem com $B = 0{,}201$ e $\omega_o = 1$.

Configuração Sallen-Key

Circuito:




$$Av(s) = \frac{G\left(\frac{\omega_o}{Q}\right) s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

$$G = \frac{\mu}{4-\mu} \longrightarrow$$

$$\mu = 1 + \frac{Rb}{Ra}$$

$$\omega_o = \frac{\sqrt{2}}{RC}$$

$$Q = \frac{\sqrt{2}}{4-K}$$

$$\mu = 4 - \frac{\sqrt{2}}{Q}$$

$(s^2 + 0{,}201s + 1)$

$\omega_o = 1 = \dfrac{\sqrt{2}}{RC}$ , $C = 1$ , $R = \sqrt{2}$

$\mu = 4 - \dfrac{\sqrt{2}}{Q}$ como $\dfrac{\omega_o}{Q} = 0{,}201$

$Q = 4{,}975 \longrightarrow K = 1 + \dfrac{Rb}{Ra} = 3{,}716$

$\dfrac{Rb}{Ra} = 2{,}716 \longrightarrow Ra = 10K$ *

$Rb = 27K\underline{1}60$ *

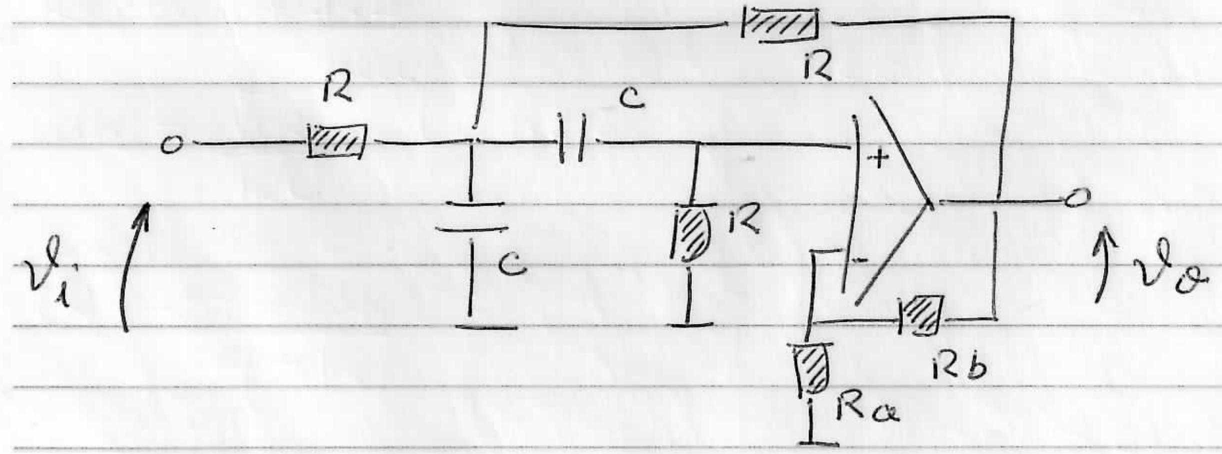
$\omega_o = 2\pi f_o = 2\pi\, 995 = 6.251\ r/s$

$\omega_o = 1 \xrightarrow[6.251]{KF} 6.251$

$C = 1 \longrightarrow 159{,}95\mu F$

$C = 159{,}9\mu F \xrightarrow[15.995]{KZ} 10nF$ *

$R = \sqrt{2} \longrightarrow R = 22.621\ \Omega$ *



# 3.2 Passa Faixa 2ª Ordem (Chebyshev)

$$\frac{1}{s + 1{,}30756}$$ , fazendo $s = \frac{s^2 + \omega_o^2}{Bs}$

teremos $$\frac{0{,}201\, s}{(s^2 + 0{,}26281 s + 1)}$$

$(s^2 + 0{,}26281 s + 1)$

$\omega_o = \frac{\sqrt{2}}{RC} \to C = 1F \to R = \sqrt{2}\ \Omega$

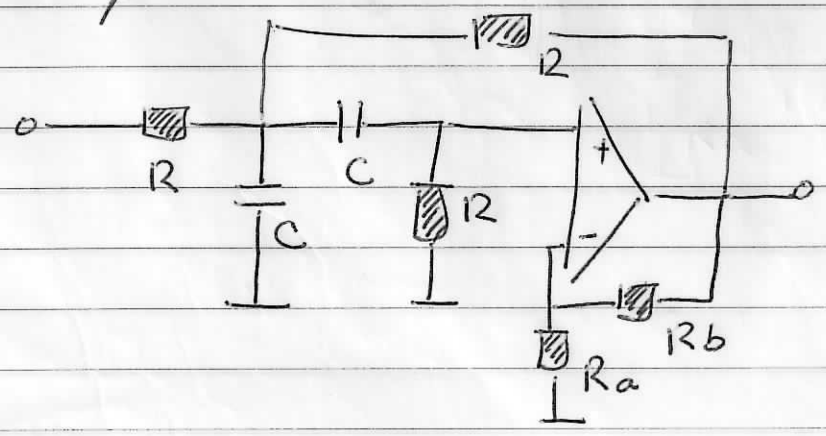
$\frac{\omega_o}{Q} = 0{,}26281 \to Q = 3{,}805$

$\mu = 4 - \frac{\sqrt{2}}{Q} = 1 + \frac{Rb}{Ra} = 3{,}628$

$\frac{Rb}{Ra} = 2{,}628 \to$ $Ra = 10k\ \Omega$ *
$Rb = 26.280\ \Omega$ *

do item 3.1 temos que $kf = 6.251$
$kz = 15.995$

e que para $C = 10n$, $R = 22.620\ \Omega$ **



4. Passa Faixa 4º Ordem $f_{c1}$ 900
$f_{c2}$ 1.100
Hz

$B = f_{c2} - f_{c1} = 200$
$f_0 = \sqrt{f_{c1} f_{c2}} = 995$ Hz $\rightarrow \omega_0 = 6.251$ r/s

$Q = \dfrac{f_0}{B} = 4{,}975$

p/ $\omega_0 = 1$, $B = \dfrac{\omega_0}{Q} = 0{,}201$

4.1 Butterworth

$(s^2 + \sqrt{2}\, s + 1) \longrightarrow \begin{matrix} s_1 = 1 \angle 135^\circ \\ s_2 = 1 \angle -135^\circ \end{matrix}$

agora resolvendo $s^2 - s_k B + \omega_0^2 = 0$

teremos:

$(s^2 - 0{,}201 \angle 135\, s + 1) \left\{ \begin{matrix} s_3 \; 0{,}931 \angle -94{,}1^\circ \\ s_4 \; 1{,}07 \angle 94{,}1^\circ \end{matrix} \right.$

$(s^2 - 0{,}201 \angle -135\, s + 1) \left\{ \begin{matrix} s_5 \; 0{,}931 \angle 94{,}1^\circ \\ s_6 \; 1{,}07 \angle -94{,}1^\circ \end{matrix} \right.$

para encontrar o polinômio de Butterworth para o Passa Faixa fazemos:

$$\big((s - s_3)(s - s_5)\big)\big((s - s_4)(s - s_6)\big)$$

e teremos:

$$(s^2 + 0{,}153\, s + 1{,}15)(s^2 + 0{,}133\, s + 0{,}8668)$$

$\rightarrow$

$$\underbrace{(s^2+0{,}153\,s+1{,}15)}_{A}\,\underbrace{(s^2+0{,}133\,s+0{,}8668)}_{B}$$

A) $\omega_o = \sqrt{1{,}15} = \dfrac{\sqrt{2}}{RC} = 1{,}0723$

$C = 1F \rightarrow R = \dfrac{\sqrt{2}}{1{,}0723} = 1{,}3188$

$\dfrac{\omega_o}{Q} = 0{,}153 \rightarrow Q = 7{,}009$

$\mu = 4 - \dfrac{\sqrt{2}}{Q} = 3{,}798 = 1 + \dfrac{R_b}{R_a}$

$\dfrac{R_b}{R_a} = 2{,}798 \longrightarrow R_a = 10K$ *

$R_b = 27.980\,\Omega$ *

$\omega = 1 \xrightarrow[6.251]{k_f} \omega = 6.251$

$C = 1 \longrightarrow C = 159{,}9\,\mu F$

$C = 159{,}9\,\mu F \longrightarrow C = 10n$ *

$R = 1{,}3188 \longrightarrow R = 21{,}097\,\Omega$ *

B) $\omega_o = \sqrt{0{,}8668} = 0{,}931 = \dfrac{\sqrt{2}}{RC}$

$C = 1F \rightarrow R = 1{,}5189$

$\dfrac{\omega_o}{Q} = 0{,}133 \rightarrow Q = 7{,}000$

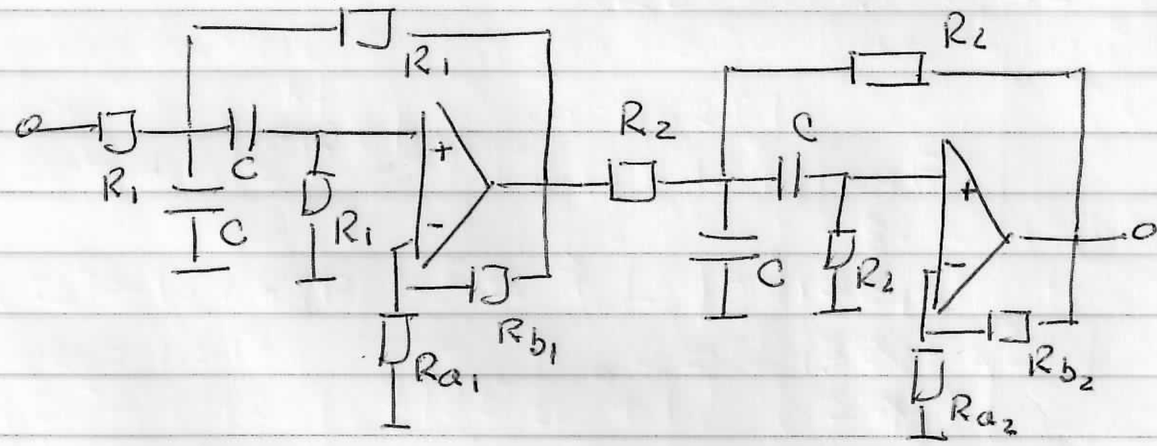
$\mu = 4 - \sqrt{2}/Q = 3{,}798 = 1 + \dfrac{R_b}{R_a}$

$\dfrac{R_b}{R_a} = 2{,}798 \longrightarrow R_a = 10k$ *

$R_b = 27.980\,\Omega$ *

utilizando kf e kz do bloco A
teremos → C = 10n *
R = 24.294 Ω **

| | Estagio 1 | Estagio 2 | |
|---|---|---|---|
| R | 21.097 | 24.294 | Ω |
| C | 10n | 10n | F |
| Ra | 10K | 10k | Ω |
| Rb | 27.980 | 27.980 | Ω |



Chebyshev, P. Faixa, 4ª Ordem.

$\alpha_p = 2\,dB$

$(s^2 + 0,803816\,s + 0,823060)$

$2 \times s_{1,2} = -0,803816 \pm \sqrt{(0,803816)^2 - 4 \times 0,823060}$

$2 s_1 = -0,803816 + j\,1,6267 \rightarrow 1,8144 \angle 116,3^\circ$

$2 s_2 = -0,803816 - j\,1,6267 \rightarrow 1,8144 \angle -116,3^\circ$

$s_1 = 0,90723 \angle 116,3^\circ = -401,9m + j\,813,3m$

$s_2 = 0,90723 \angle -116,3 = -401,9m - j\,813,3m$

$s_1 = a + jb$

$s_2 = a - jb$

$(s^2 - s_1 B s + \omega_0^2)$ (1)

$(s^2 - s_2 B s + \omega_0^2)$ (2)

$f_0 = 995 \rightarrow \omega_0 = 6.252$

$B = f_{c_2} - f_{c_1} = 200$

$Q = f_0 / B = 4.975$

se $\omega_0 = 1$, $B = \dfrac{\omega_0}{Q}$

$B = 0,201$

(1) $(s^2 - (a + jb)Bs + \omega_0^2 = 0$

$2 s_{3,4} = +(a+jb)B \pm \sqrt{((a+jb)B)^2 - 4\omega_0^2}$

$= (a+jb)B \pm \sqrt{(a^2 + 2jab - b^2)B^2 - 4}$

$= (a+jb)B \pm \sqrt{(a^2 - b^2)B^2 + j2abB^2 - 4}$

$2 s_{2,4} = (a+jb)B \pm \sqrt{(a^2 - b^2)B^2 - 4 + j2abB^2}$

$(s^2 - 182,3m \angle 116,3^\circ\, s + 1)$ (1)

$(s^2 - 182,3m \angle -116,3^\circ\, s + 1)$ (2)

Resolvendo (1) temos:

$$2\,s_{3,4} = +182{,}3m \angle 116{,}3^\circ \pm \sqrt{(-182{,}3m \angle 116{,}3^\circ)^2 - 4}$$

$$= 182{,}3m \angle 116{,}3^\circ \pm \sqrt{33{,}25m \angle 232{,}6 - 4}$$

$$2\,s_{3,4} = -80{,}77m + j163{,}4m \pm \sqrt{-20{,}2m - j26{,}4m - 4}$$

$$2\,s_{3,4} = -80{,}77m + j163{,}4m \pm \sqrt{-4{,}020 - j26{,}4m}$$

$$2\,s_{3,4} = -80{,}77m + j163{,}4m \pm \sqrt{4{,}0203 \angle -179{,}62^\circ}$$

$$2\,s_{3,4} = -80{,}77m + j163{,}4m \pm 2{,}0051 \angle -89{,}81^\circ$$

$$2\,s_{3,4} = -80{,}72m + j163{,}4m \pm (6{,}649m - j2{,}0051)$$

$$-74{,}12m - j1{,}8417 = 1{,}843 \angle -92{,}3^\circ / 2$$

$$2\,s_{3,4} = -87{,}42m + j2{,}1685 = 2{,}170 \angle 92{,}3^\circ / 2$$

Resolvendo (2) temos:

$$2\,s_{5,6} = 182{,}3m \angle -116{,}3^\circ \pm \sqrt{(-182{,}3m \angle -116{,}3^\circ)^2 - 4}$$

$$2\,s_{5,6} = 182{,}3m \angle -116{,}3^\circ \pm \sqrt{4{,}0203 \angle +179{,}62^\circ}$$

$$2\,s_{5,6} = 182{,}3m \angle -116{,}3^\circ \pm 2{,}0051 \angle +89{,}81^\circ$$

$$2\,s_{6,6} = -80{,}72m - j163{,}4m \pm (6{,}649m + j2{,}0051)$$

$$-74{,}12m + j18417 = 1{,}843 \angle 92{,}3^\circ$$

$$2\,s_{5,6} = -87{,}42m - j2{,}1685 = 2{,}170 \angle -92{,}3^\circ / 2$$

$s_3 = -37,06m - j920,8m = 921,5m \angle -92,3^\circ$

$s_4 = -43,71m + j1.084 = 1.085 \angle 92,3^\circ$

$s_5 = -37,06m + j920,8m = 921,5m \angle +92,3^\circ$

$s_6 = -43,71m - j1.084 = 1.085 \angle -92,3^\circ$

$$\underbrace{(s-s_3)(s-s_5)}_{A} \quad \underbrace{(s-s_4)(s-s_6)}_{B}$$

$s_k = (a \pm jb)$

$(s-(a+jb))(s-(a-jb))$

$s^2-(a-jb)s-(a+jb)s+(a+jb)(a-jb)$

$s^2 - as + \cancel{jbs} - as - \cancel{jbs} + a^2 - \cancel{jab} + \cancel{jab} + b^2$

$(s^2 - 2as + (a^2+b^2))$, então:

$(s^2 + 74,12m\, s + 0,849)$ (A)

$(s^2 + 87,42m\, s + 1,1772)$ (B)

(A) × (B) é o polinômio de Butterworth para o Filtro passa Faixa com $\omega_0 = 1$ e $B = 0,201$

Resolvendo Bloco A

$\omega_0 = \sqrt{0,849} = 0,921 = \frac{\sqrt{2}}{RC}$

p/ $C = 1F$, $R = 1,5348$

$\frac{\omega_0}{Q} = 74,12m \rightarrow Q = \frac{0,921}{74,1m} = 12,42$

$\rightarrow$

$\mu = 4 - \frac{\sqrt{2}}{Q} = 3{,}886 = 1 + \frac{Rb}{Ra}$

$\frac{Rb}{Ra} = 2{,}886 \rightarrow Ra = 10k$ *

$Rb = 28.886k$ *

$\omega_o = 1 \quad \xrightarrow{kf = 6.251} \quad \omega_o = 2 \times \pi \times 995$

$C = 1 \qquad\qquad = 6.251$

$C = 159{,}95\mu F$

$C = 159{,}95\mu \quad \xrightarrow{kz\ 15.995} \quad C = 10nF$ *

$R = 1{,}5347 \qquad\qquad R = 24.549\,\Omega$ *

Resolução Bloco B

$s^2 + 87{,}42m\,s + 1{,}177)$

$\omega_o = \sqrt{1{,}177} = 1{,}084 = \frac{\sqrt{2}}{RC}$

p/ $C = 1F \rightarrow R = 1{,}3035$

$\frac{\omega o}{Q} = 87{,}42m$ , $Q = \frac{1{,}084}{87{,}42m} = 12{,}40$

$\mu = 4 - \frac{\sqrt{2}}{Q} = 3{,}886 = 1 + \frac{Rb}{Ra}$

$\frac{Rb}{Ra} = 2{,}886 \rightarrow Ra = 10k$ *

$Rb = 28.886k$ *

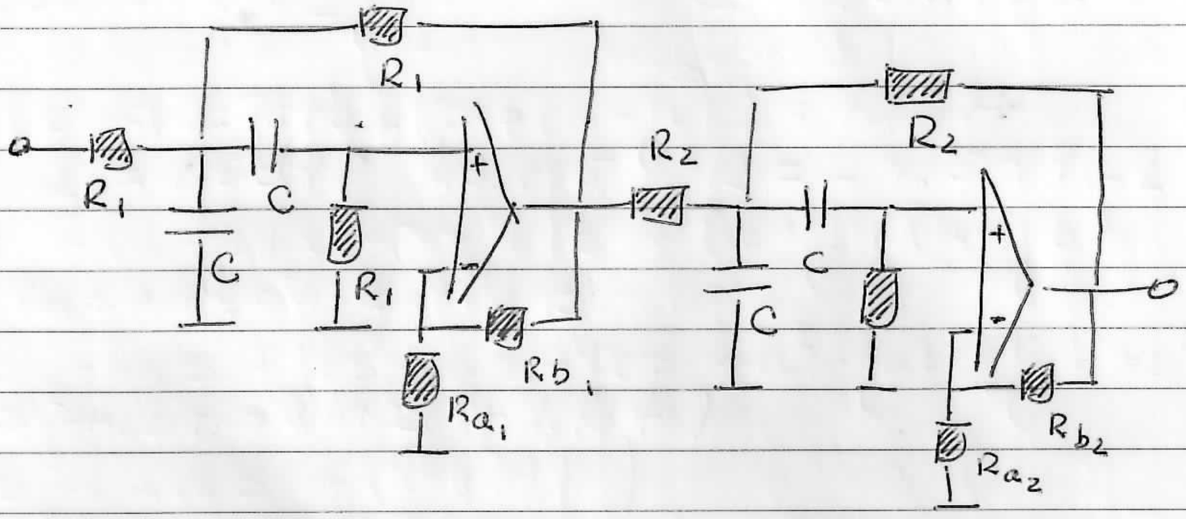
$\omega_o = 1 \quad \xrightarrow{kf_{6.251}} \quad \omega_o = 6.251$

$C = 1 \qquad\qquad C = 159{,}9\mu F$

$C = 159{,}9\mu F \quad \xrightarrow{kz\ 15.995} \quad C = 10nF$ *

$R = 1{,}3035 \qquad\qquad R = 20.842\,\Omega$ *

| | 1º Estagio | 2º Estágio |
|---|---|---|
| C | 10n | 10n |
| R | 24.549 ≅ 100k // 33k = 24.812 | 20.842 ≅ 42k // 42k |
| Ra | 10k | 10k |
| Rb | 28.886 ≃ 27k + 1k8 28.800 | 28.886 |

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Filtro P. Alta e P. B. 5º Ordem.

$f_c = 500\ Hz$

1) P.B Buther

$$\underbrace{(s^2+0{,}618034 s+1)}_{1}\underbrace{(s^2+1{,}618034 s+1)}_{2}\underbrace{(s+1)}_{3}$$

1.1) $\omega_0 = \sqrt{1} = 1$ , $\frac{\omega_0}{Q} = 0{,}618034 = \frac{1}{Q}$

$C_1 = C_2 = 1$ , $R_1 = R_2 = R = \frac{1}{\omega_0} = 1$

$\frac{Rb}{Ra} = 2 - \frac{1}{Q} = 1{,}38199 \rightarrow$ 13.819,9 Ω

$\rightarrow$ 10k

$kf = 2\pi f_c = 2\pi 500$ $C = 1 \rightarrow \frac{1}{Kf} \rightarrow$ 10n

$R = 1 \rightarrow$ 31.831

1.2) $\omega_0 = \sqrt{1} = 1$ , $\frac{1}{Q} = 1{,}618034$

$C_1 = C_2 = 1$ , $R_1 = R_2 = R = \frac{1}{\omega_0} = 1$

$\frac{Rb}{Ra} = 2 - \frac{1}{Q} = 0{,}38196 \rightarrow$ 3K820

$\rightarrow$ 10k

C = 10n

R = 31831 K

1.3 $\omega_c = 1 = \frac{1}{RC}$ se $c = 1$ , $R = 1$

$\frac{1}{R}$ → 10n

→ 31.831 Ω

2. PB Chebyshev. $f_c = 500\,Hz$

$(s^2 + 0{,}109720\,s + 0{,}936025)(s^2 + 0{,}287250\,s + 0{,}377009)$
$\times(s + 0{,}177530)$

2.1 $\omega_0 = \sqrt{b_0} = 0{,}967484$ , $\omega_0 = b_1$
$Q = \frac{\omega_0}{b_1} = 8{,}8177$ , $\frac{1}{Q} = 0{,}113407$

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = 1{,}88659 \rightarrow \boxed{\frac{18k866}{10k}}$

$C_1 = C_2 = 1$ , $R_1 = R_2 = R = \frac{1}{\omega_0} = 1{,}0336088$

$k_f = 2\pi f = 3.141{,}59$

$C = 1 \rightarrow 10\,n$
$R = 1{,}0336 \rightarrow 32.900{,}79\,\Omega$

2.2 $\omega_0 = \sqrt{b_0} = 0{,}61401058$ , $Q = \frac{\omega_0}{b_1} = 2{,}13754$
$\frac{1}{Q} = 0{,}467825$

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = 1{,}5321 \rightarrow \boxed{\begin{array}{l} R_B = 15k321 \\ R_A = 10k \end{array}}$

$C_1 = C_2 = 1$ , $R_1 = R_2 = R = \frac{1}{\omega_0} = 1{,}6286$

$C = 1 \rightarrow C = 10\,n$
$R = 1{,}6286 \rightarrow R = 51k840$

2.3 $\omega_c = b_0 = 0{,}177530 = \frac{1}{RC}$ se $C = 1$

$R = 5{,}632 \rightarrow 179.299\,\Omega$
$C = 1 \rightarrow 10\,n$

3)

P.A. Butherworth $\qquad f_c = 500\ Hz$

$$\underbrace{(s^2 + 0{,}6180341\,s + 1)}_{1}\underbrace{(s^2 + 1{,}6180341\,s + 1)}_{2}\underbrace{(s+1)}_{3}$$

3.1 $\omega_0 = \frac{1}{\sqrt{1}} = 1 \qquad \frac{1}{Q} = \frac{b_1}{\sqrt{b_0}} = 0{,}6180341$ $\quad$ * ver pag. 19

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = \frac{13.819{,}66}{10K}$ $\qquad k_f = 2\pi f$

$k_z = 31.830$

$C_1 = C_2 = 1 \qquad R_1 = R_2 = R = \frac{1}{\omega_0} = 1$

$C = 1 \qquad \boxed{\frac{10\mu}{31.830}}$

$R = 1$

3.2 $\omega_0 = \frac{1}{\sqrt{1}} = 1 \qquad \frac{1}{Q} = \frac{b_1}{\sqrt{b_0}} = 1{,}6180341$ $\quad$ * ver pag. 19

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = 0{,}381966 \rightarrow \boxed{\frac{3K819{,}66}{10K}}$

$C_1 = C_2 : 1 \qquad R = \frac{1}{\omega_0} = 1$

$C \qquad \boxed{\frac{10}{31.830}}$

$R = 1$

3.3 $\omega_c = \frac{1}{RC} \rightarrow C = 1 \qquad R = \frac{1}{\omega_c} = 1$

$C \qquad \boxed{\frac{10\mu}{31.830}}$

$R = 1$

4. P.A Chebyshev 5º Ordem 3dB

$\underbrace{(s^2+0{,}10972\,s+0{,}936025)}_{1}\underbrace{(s^2+0{,}28725\,s+0{,}377009)}_{2}\underbrace{(s+0{,}17753)}_{3}$

4.1 $\omega_0 = \frac{1}{\sqrt{b_0}} = 1{,}0336 \rightarrow \frac{1}{Q} = \frac{b_1}{\sqrt{b_0}} = 0{,}11341$

$Q = 8{,}8177$

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = 1{,}8866 \rightarrow \frac{18k866}{10k}$

$C_1 = C_2 = 1$, $R_1 = R_2 = R = \frac{1}{\omega_0} = 0{,}96748$ $k_f = 2\pi f$

$C \rightarrow 10n$ $k_z =$

$R = 0{,}96741 \rightarrow 30.795{,}9\,\Omega$

4.2 $\omega_0 = \frac{1}{\sqrt{b_0}} = 1{,}62863 \rightarrow \frac{1}{Q} = \frac{b_1}{\sqrt{b_0}} = 0{,}4678$

$Q = 2{,}137$

$\frac{R_b}{R_a} = 2 - \frac{1}{Q} = \frac{15.322}{10.000}$

$C_1 = C_2 = 1$, $R_1 = R_2 = R = \frac{1}{\omega_0} = 0{,}614013$

$C = 1 \rightarrow 10n$

$R = 0{,}614 \rightarrow 19.545\,\Omega$

4.3 $\omega_c = \frac{1}{b_0} = \frac{1}{RC}$ se $C = 1$

$R = b_0 = 0{,}17753$

$C \rightarrow 10n$

$R = 0{,}17753 \rightarrow 5.651\,\Omega$

Projetos com
o controle
do
ganho

# Calculos de Filtros usando os polinomios normalizados de Butterworth e Chebyshev:

## Exemplo 1

Calcule o filtro com as seguintes caracteristicas: P. Baixa 5ª Ordem Chebyshev

Polinômios:

$(s^2 + 0{,}109720s + 0{,}936025)$ A

$(s^2 + 0{,}287250s + 0{,}377009)$ B

$(s + 0{,}177530)$ C

$f_{corte} = 2.500\ Hz$

Ondulacão 3 dBs.

$G = 1{,}1$ /estágio de 2ª Ordem.

Estagio A

$$w_0 = \sqrt{0{,}936025} = 0{,}9674838$$

$$\frac{w_0}{Q} = 0{,}109720 \Rightarrow Q = 8{,}8177$$

Fazemos $C_1 = 1$ e $R_2 = 1$

$$\frac{R_b}{R_a} = 0{,}1 = \frac{10k}{100k}$$

$$C_2 = (\mu - 1)\,C_1 = 0{,}1$$

$$R_1 = Q^2(\mu-1)R_2 = 8{,}8^2(0{,}1) = 7{,}77527$$

$$w_0' = \frac{1}{\sqrt{1 \times 0{,}1 \times 7{,}775 \times 1}} = 1{,}1341$$

(Desloc. 1) $\xrightarrow{\times k_Z}$

$w_0 = 1{,}1341 \quad 0{,}9674838$

$C_1 = 1 \xrightarrow{\times \frac{1}{k_Z}} 1{,}1722$

$C_2 = 0{,}1 \longrightarrow 0{,}011722$

Deslocamento Z

$w_c = 1 \qquad 2\pi 2500$

$C_1 = 1{,}1722 \longrightarrow 10\ nF$

$C_2 = 0{,}11722 \longrightarrow 1\ nF$

$R_1 = 7{,}77 \longrightarrow 58.022\ \Omega$

$R_2 = 1 \longrightarrow 7.462\ \Omega$

Estagio B

$\omega o = \sqrt{0,377009} = 0,61401$

$\frac{\omega o}{Q} = 0,287250 \Rightarrow Q = 2,13754$

$\frac{Rb}{Ra} = 1,1 - 1 = 0,1 = \frac{10k}{100k}$

Fazemos $C_1 = 1$ e $R_2 = 1$

$C_2 = (\mu - 1) C_1 = 0,1$

$R_1 = Q^2 (\mu - 1) R_2 = 0,45691$

$\omega o = \frac{1}{\sqrt{1 \times 0,1 \times 0,45691 \times 1}} = 4,6782$

Desloc. 1

$\omega o = 4,6782 \rightarrow 0,61401$

$C_1 = 1 \rightarrow 7,6191$

$C_2 = 0,1 \rightarrow 0,76191$

Deslocamento 2:

$\omega c = 1 \quad 2\pi 2500$

$C_1 = 7,6191 \rightarrow 10n$

$C_2 = 0,76191 \rightarrow 1n$

$R_1 = 0,45691 \rightarrow 22.085$

$R_2 = 1 \rightarrow 48.335$

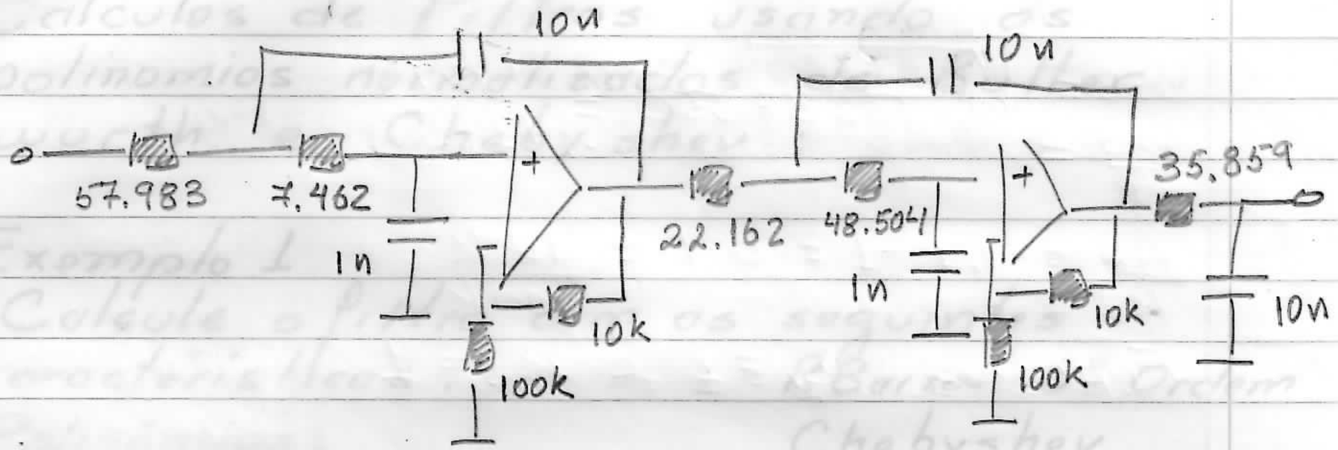
Estágio C

$\omega o = \frac{1}{RC} = 0,177530 \rightarrow C = 1, \; R = 5,6328$

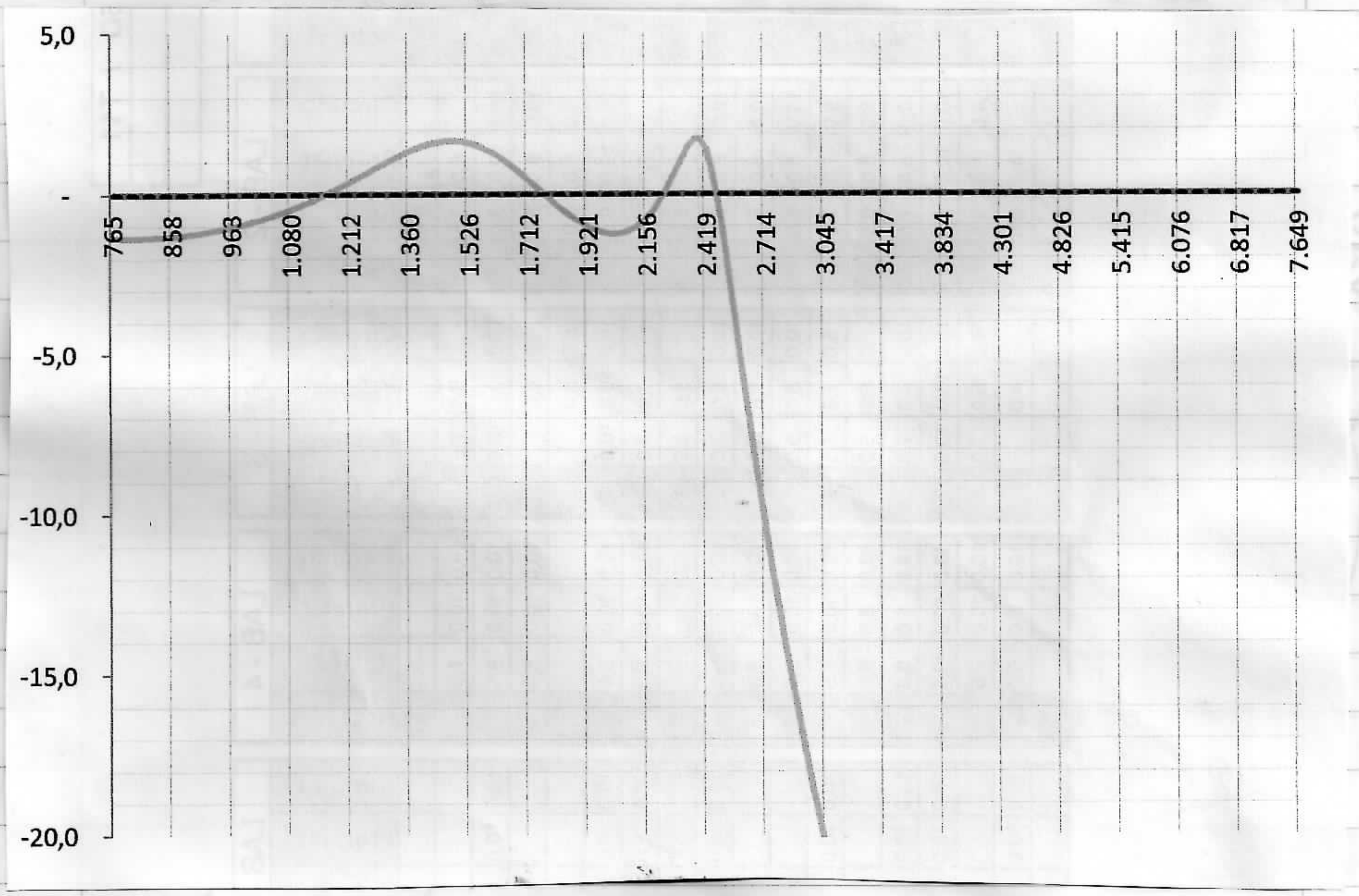
$\omega c = 1 \quad 2\pi 2500$

$C = 1 \rightarrow 10n$ ✓

$R = 5,6328 \rightarrow 35.859$


10n
10n
57.983
7.462
1n
100k
10k
22.162
48.504
1n
10k
100k
35.859
10n

Resultado:


5,0
-
-5,0
-10,0
-15,0
-20,0
765
858
963
1.080
1.212
1.360
1.526
1.712
1.921
2.156
2.419
2.714
3.045
3.417
3.834
4.301
4.826
5.415
6.076
6.817
7.649

Exemplo 2.

Calcule o seguinte filtro:

P. Faixa 6º Ordem

$f_{c1} = 1600\,Hz$

$f_{c2} = 1.900\,Hz$

Polinômios:

A $(s^2 + 0{,}298620\,s + 0{,}839174)$ $G = 5{,}7$ /Estágio

B $(s + 0{,}298.620)$

$f_0 = 1.743{,}56$

$B = 300$ $/ = 0{,}17206/_{w_0 = 1}$

$Q = f_0/B = 5{,}812$

Parte A

$B = 0{,}172061759$ p/ $\omega_0 = 1$

$(s^2 + 0{,}298620\,s + 0{,}839174)$

$$s_{1,2} = \frac{-0{,}298620 \pm \sqrt{(0{,}298620)^2 - 4 \times 0{,}839174}}{2}$$

$$s_{1,2} = \frac{-0{,}298620}{2} \pm j\,\frac{1{,}80762886}{2}$$

$$s_{1,2} = -0{,}14931 \pm j\,0{,}90381443 = 0{,}916064408 \angle \pm 99{,}3805$$

6387

resolvendo $s^2 - s_k B s + \omega_0^2$ teremos

$$s^2 - 0{,}157619653 \angle 99{,}3805\, s + 1 \rightarrow s_3,\ s_4$$

$$s^2 - 0{,}157619653 \angle -99{,}3805\, s + 1 \rightarrow s_5,\ s_6$$

$$s_{3,4} = \frac{+0{,}15762 \angle 99{,}381 \pm \sqrt{(0{,}15762 \angle 99{,}381)^2 - 4}}{2}$$

$$s_{34} = 0{,}07881 \angle 99{,}381 \pm 0{,}5 \sqrt{0{,}024844 \angle 198{,}762 - 4}$$

$$= -0{,}012846 + j\,0{,}077756 \pm 0{,}5 \sqrt{-0{,}023524 - j\,0{,}0079907 - 4}$$

$$= -0{,}012846 + j\,0{,}077756 \pm 0{,}5 \sqrt{-4{,}023524 - j\,0{,}0079907}$$

$$= -0{,}012846 + j\,0{,}077756 \pm 0{,}5 \sqrt{4{,}02353 \angle -179{,}89}$$

$s_{3,4} = -0{,}012846 + j0{,}077756 \pm 0{,}5\ (2{,}005873\ \angle -89{,}943)$

$s_{3,4} = -0{,}012846 + j0{,}077756 \pm (1{,}0029369\ \angle -89{,}943)$

$s_{3,4} = -0{,}012846 + j0{,}077756 \pm 0{,}0009977 - j1{,}002936$

$s_3 = -0{,}011848 - j0{,}92518$

$s_4 = -0{,}013844 + j1{,}080692$

$s_{5,6} = +\dfrac{0{,}15762}{2} \angle -99{,}3805 \pm 0{,}5\sqrt{(0{,}15762 \angle -99{,}3805)^2 - 4}$

$s_{5,6} = +0{,}07881 \angle -99{,}3805 \pm 0{,}5\sqrt{-0{,}02352 + j0{,}0079903 - 4}$

$s_{5,6} = +0{,}07881 \angle -99{,}3805 \pm 0{,}5\sqrt{-4{,}02352 + j0{,}0079903}$

$s_{5,6} = +0{,}07881 \angle -99{,}3805 \pm 0{,}5\sqrt{4{,}02353 \angle 179{,}886}$

$s_{5,6} = +0{,}07881 \angle -99{,}3805 \pm 0{,}5 \times (2{,}005873\ \angle 89{,}943)$

$s_{5,6} = -0{,}012845 - j0{,}077756 \pm 1{,}0029369 \angle 89{,}943$

$s_{5,6} = -0{,}012845 - j0{,}077756 \pm (+0{,}00099776 + j1{,}0029364)$

$s_5 = -0{,}011847 + j0{,}9251 = s_3^*$

$s_6 = -0{,}013843 - j1{,}08069 = s_4^*$

$(s - \overbrace{(a+jb)}^{s_3})(s - \overbrace{(a-jb)}^{s_5})$ resulta:

$s^2 - 2as + (a^2+b^2)$

de $s_3$ e $s_5$ temos:

$(s + 0{,}023694\,s + 0{,}8559)$ e

de $s_4$ e $s_6$ temos

$(s + 0{,}02769\,s + 1{,}16808)$

Parte B $\longrightarrow$

$$B \to \frac{1}{(s + b_0)} = \frac{1}{s - sk} \quad \text{onde } sk = -b_0$$

substituindo $s = \dfrac{s^2 + w_0^2}{Bs}$ temos

$$\frac{1}{\dfrac{s^2 + w_0}{Bs} - sk} = \frac{Bs}{s^2 - skBs + w_0^2}$$

para este caso temos

$sk = -b_0 = -0,298620$

$B = 0,172061$ p/ $w_0 = 1$ e então:

$$\frac{0,1720\,s}{(s^2 + 0,051365 + 1)} \leftarrow$$

Finalmente Temos o Polinômio de Chebychev para o filtro Passa Faixa de 6º Ordem com $B = 0,1720$ e $w_0 = 1$:

$(s^2 + 0,023694\,s + 0,8559)$ Estágio 1

$(s^2 + 0,027695 + 1,16808)$ " 2

$(s^2 + 0,051365 + 1)$ " 3

1. Estágio 1 $G = 5,7$, $a = \dfrac{C_2}{C_1} = 0,22$, $C_1 = 1$

e $R_2 = 1$

$w_0 = \sqrt{0,8559} = 0,9251$

$\dfrac{w_0}{Q} = 0,023694 \Rightarrow Q = 39,0456$

$\longrightarrow$

$$G = \mu = 5{,}7 = 1 + \frac{Rb}{Ra} \qquad \frac{Rb}{Ra} = \frac{47\,k}{10\,k}$$

$$R_3 = \frac{a+1}{a} \times \frac{1}{\mu - 1} \times R_2 = 1{,}17988$$

$$R_1 = \frac{-R_2 + \sqrt{R_2^2 + 4aQ^2 R_2 R_3}}{2} = 19{,}39$$

com estes valores calculamos

$$\omega_0 = \sqrt{\frac{R_1 + R_L}{R_1 R_L R_3 C_1 C_2}} = 2{,}01$$

que deverá ser transportado para 0,9251

$k_f = 0{,}4595$

$\omega_0 = 2{,}01 \longrightarrow 0{,}9251$

$C = 1 \longrightarrow 2{,}176$

$C = 0{,}22 \longrightarrow 0{,}4787$

Deslocamentos

$\omega_0 = 1 \longrightarrow 2\pi\, 1.743$

$C_1 = 2{,}176 \longrightarrow 10\ nF$ $C_1$

$C_1 = 0{,}4787 \longrightarrow 2{,}2\ nF$ $C_2$

$R_2 \xrightarrow{k_z = 19\,869} 19.869$ $R_2$

$R_3 \longrightarrow 23.447$ $R_3$

$R_L \longrightarrow 385.264$ $R_L$

Estágio 2 $\quad G = 5{,}7\ ,\ a = \frac{C_2}{C_1} = 0{,}1\ ,\ C_1 = 1$

$C_2 = 0{,}1\ ,\ R_2 = 1$

$$\omega_0 = \sqrt{1{,}16808} = 1{,}080777$$

$$\frac{\omega_0}{Q} = 0{,}02769 \Rightarrow Q = 39{,}03$$

$$\frac{Rb}{Ra} = G - 1 = \frac{47K}{10K}$$

$$R_3 = \frac{a+1}{a} \frac{1}{\mu - 1} \times R_2 = 2,34$$

$$R_1 = \frac{-R_2 + \sqrt{R_2^2 + 4aQ^2R_2R_3}}{2} = 18,3868$$

$$\omega_0 = \sqrt{\frac{R_1 + R_2}{R_1R_2R_3\ C_1C_2}} = 2,1227$$

Transportando de volta p/ a $\omega_0$ do polinômio Temos:

$\omega_0 = 2,1222 \longrightarrow \omega_0 = 1,080777$

$C_1 = 1 \longrightarrow 1,96411$

$C_2 = 0,1 \longrightarrow 0,196411$

Deslocamentos

$\omega_0 = 1 \longrightarrow 2\pi\ 1,7413$

$C_1 = 1,96411 \longrightarrow 10n \quad C_1$

$C_2 = 0,196411 \longrightarrow 1n \quad C_2$

$R_2 = 1 \longrightarrow 17,9341 \quad R_2$

$R_3 = 2,34 \longrightarrow 41,966 \quad R_3$

$R_1 = 18,38 \longrightarrow 329,626 \quad R_1$

Estágio 3

$G = 5,7, \quad a = C_1/C_2 = 0,1$

$C_1 = 1, \ C_2 = 0,1$ e $R_2 = 1$, do polinômio temos:

$$\omega_0 = \sqrt{1} = 1$$

$$\frac{\omega_0}{Q} = 0,05136 \Rightarrow Q = 19,47$$

$\longrightarrow$

$$\frac{R_b}{R_a} = G - 1 = \frac{47}{10k}$$

$$R_3 = \frac{a+1}{a} \times \frac{1}{u-1} \quad R_2 = 2,34$$

$$R_1 = \frac{-R_2 + \sqrt{R_2^2 + 4a\,Q^2 R_2 R_3}}{2} = 8,93$$

$$\omega_o = \sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3\, C_1 C_2}} = 2,179$$

deslocando para a frequência do polinômio temos:

$\omega_o = 2,129 \rightarrow 1$

$C_1 = 1 \rightarrow 2.179$

$C_2 = 0,1 \rightarrow 0,2179$

Deslocamentos

$\omega_o = 1 \rightarrow 2\pi\, 1.743$

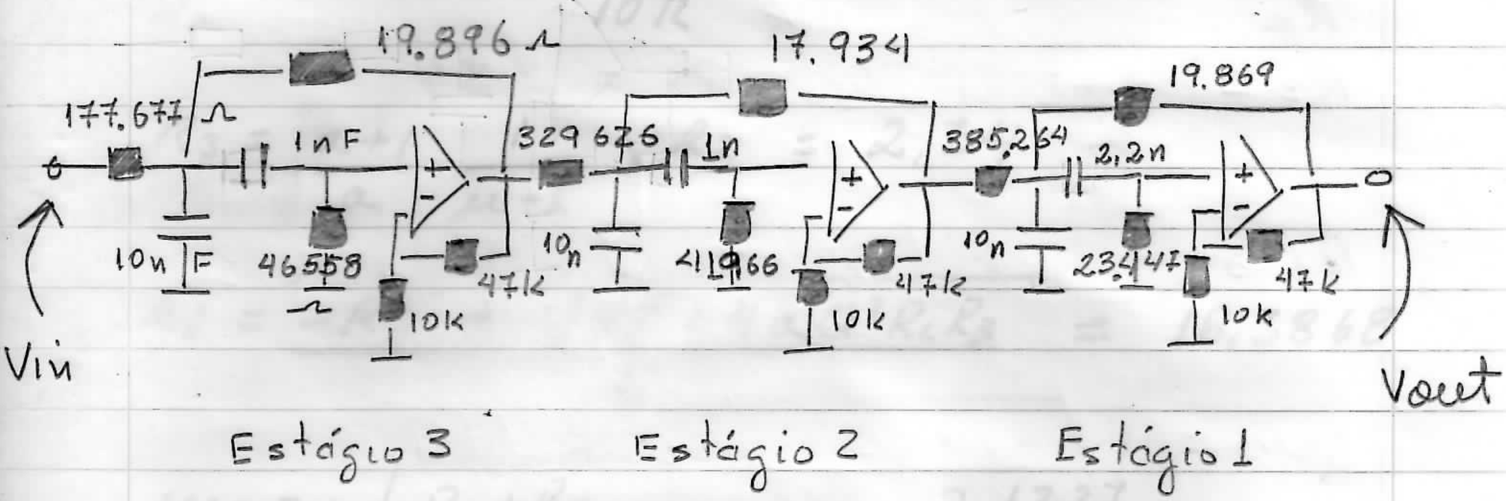
$C_1 = 2.179 \rightarrow 10n$

$C_2 = 0,2179 \rightarrow 1n$

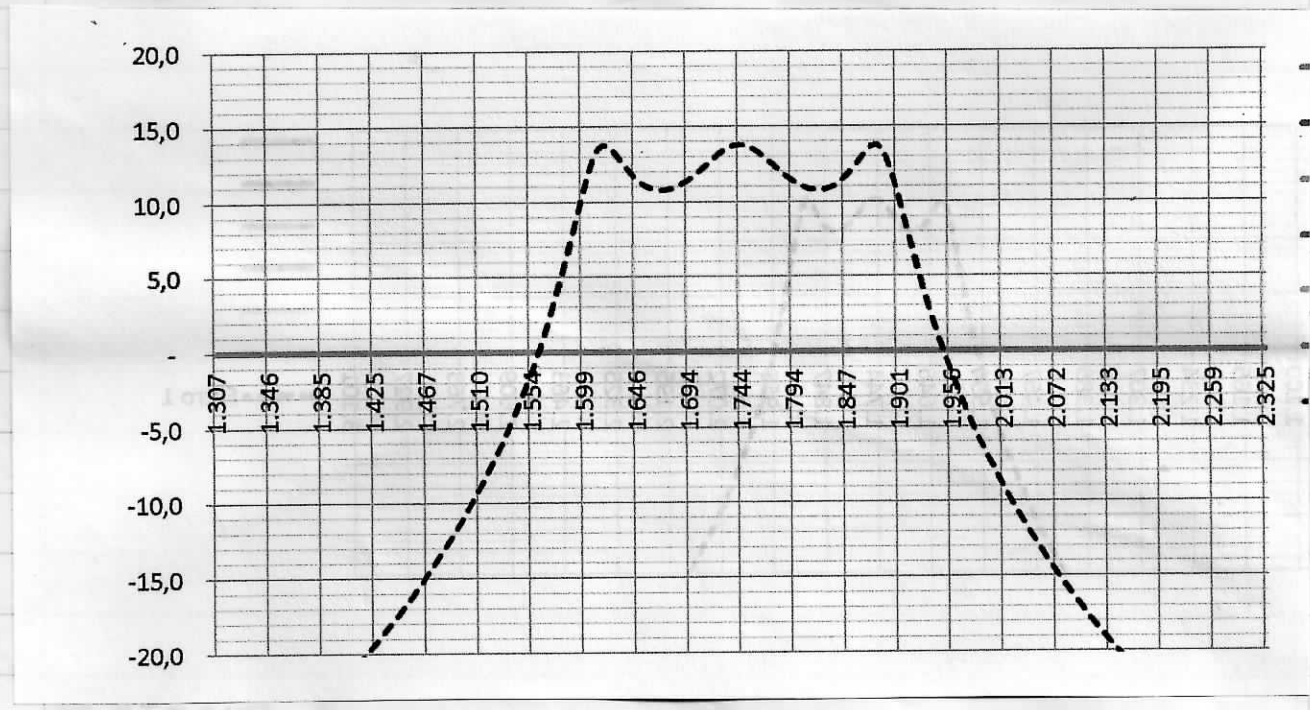
$R_2 = 1 \rightarrow 19.896$

$R_3 = 2,34 \rightarrow 46.558$

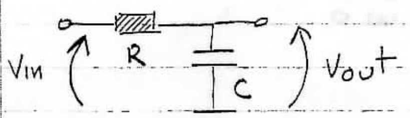
$R_1 = 8,93 \rightarrow 177,677$



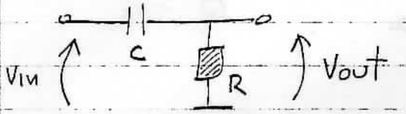

Resultado Final:



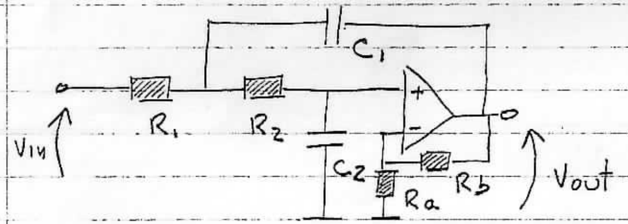
$$s = j\omega$$



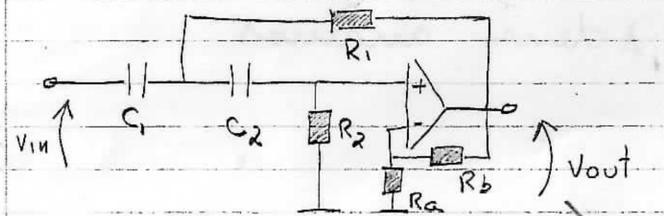

$$Av(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}} \times \frac{\omega_c}{\omega_c} \qquad Av(s) = \frac{\omega_c}{s + \omega_c} \qquad \omega_c = \frac{1}{RC}$$



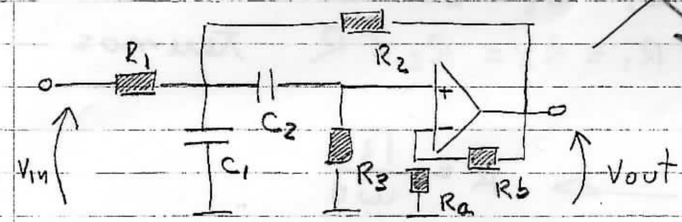

$$Av(j\omega) = \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}} \times \frac{\omega_c}{\omega_c} \qquad Av(s) = \frac{s}{s + \omega_c} \qquad \omega_c = \frac{1}{RC}$$




$$Av(j\omega) = G\,\frac{1}{\left(j\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\frac{\omega}{\omega_0} + 1} \times \frac{\omega_0^2}{\omega_0^2}\,, \quad Av(s) = G\,\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$




$$Av(j\omega) = G\,\frac{j\frac{1}{Q}\frac{\omega}{\omega_0}}{\left(j\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\frac{\omega}{\omega_0} + 1} \times \frac{\omega_0^2}{\omega_0^2}\,, \quad Av(s) = G\,\frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$




$$Av(j\omega) = G\,\frac{\left(j\frac{\omega}{\omega_0}\right)^2}{\left(j\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\frac{\omega}{\omega_0} + 1} \times \frac{\omega_0^2}{\omega_0^2}\,, \quad Av(s) = G\,\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

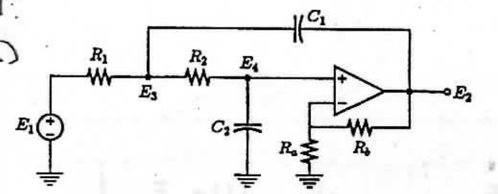



Figure 10.1: The Sallen-Key lowpass biquad.

$$\frac{E_2}{E_1} = \frac{\frac{\mu}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-\mu}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

where $\mu = 1 + \frac{R_b}{R_a}$.

Using the standard notation

$$H_{LP}(s) = \frac{G\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

we have

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q = \frac{\frac{1}{\sqrt{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-\mu}{R_2C_2}}$$

$$G = \mu$$

Se fizermos $C_1 = C_2 = 1F$ e $R_1 = R_2 = R$ teremos:

$$\omega_0 = \frac{1}{R} \longrightarrow R = \frac{1}{\omega_0}$$

$$\left.\begin{array}{l} Q = \dfrac{1}{3-\mu} \\ \mu = 1 + \dfrac{R_b}{R_a} \end{array}\right\} \longrightarrow \frac{R_b}{R_a} = 2 - \frac{1}{Q}$$

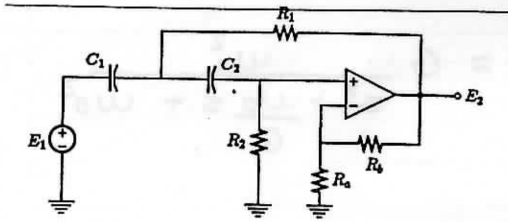



Figure 10.3: The Sallen-Key highpass biquad.

$$\frac{E_2}{E_1} = \frac{\mu s^2}{s^2 + \left(\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{1-\mu}{R_1C_1}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

Using the standard expression

$$H_{HP}(s) = \frac{Gs^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

we can identify the following relationships:

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q = \frac{\frac{1}{\sqrt{R_1R_2C_1C_2}}}{\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{1-\mu}{R_1C_1}}$$

$$G = \mu$$

Idem anteriores

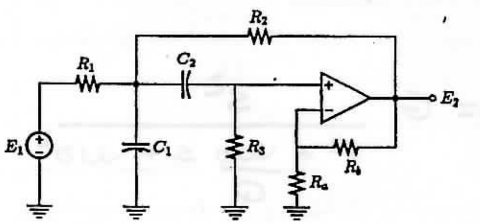



Figure 10.4: A Sallen-Key bandpass biquad.

$$\frac{E_2}{E_1} = \frac{\frac{\mu}{R_1C_1}s}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-\mu}{R_2C_1}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

$$H_{BP}(s) = \frac{G\left(\frac{\omega_0}{Q}\right)s}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

we readily identify the following expressions for the three parameters:

$$\omega_0 = \sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

$$Q = \frac{\sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-\mu}{R_2C_1}}$$

$$G = \frac{\frac{\mu}{R_1C_1}}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-\mu}{R_2C_1}}$$

Fazendo $C_1 = C_2 = 1F$ e $R_1 = R_2 = R_3 = R$ teremos:

$$\omega_0 = \frac{\sqrt{2}}{R} \longrightarrow R = \frac{\sqrt{2}}{\omega_0}$$

$$\left.\begin{array}{l} Q = \dfrac{\sqrt{2}}{4-\mu} \\ \mu = 1 + \dfrac{R_b}{R_a} \end{array}\right\} \longrightarrow \begin{array}{l} \dfrac{R_b}{R_a} = 3 - \dfrac{\sqrt{2}}{Q} \\ G = \dfrac{\mu}{4-\mu} \end{array}$$

Table A.1. **Butterworth polynomials in factored form**

| $n$ | Polynomials |
|---|---|
| 1 | $s+1$ |
| 2 | $s^2+1.414214s+1$ |
| 3 | $(s^2+s+1)(s+1)$ |
| 4 | $(s^2+0.765367s+1)(s^2+1.847759s+1)$ |
| 5 | $(s^2+0.618034s+1)(s^2+1.618034s+1)(s+1)$ |
| 6 | $(s^2+0.517638s+1)(s^2+1.414214s+1)(s^2+1.931852s+1)$ |
| 7 | $(s^2+0.445042s+1)(s^2+1.246980s+1)(s^2+1.801938s+1)(s+1)$ |
| 8 | $(s^2+0.390181s+1)(s^2+1.111140s+1)(s^2+1.662939s+1)(s^2+1.961571s+1)$ |
| 9 | $(s^2+0.347296s+1)(s^2+s+1)(s^2+1.532089s+1)(s^2+1.879385s+1)(s+1)$ |
| 10 | $(s^2+0.312869s+1)(s^2+0.907981s+1)(s^2+1.414214s+1)(s^2+1.782013s+1)$<br>$\times(s^2+1.975377s+1)$ |

Table A.3. **Denominator polynomials in factored form for Chebyshev filters**

$\alpha_p = 0.1$ dB $\quad (\epsilon^2 = 0.0232930 \quad \epsilon = 0.152620)$

| $n$ | Polynomials |
|---|---|
| 1 | $s+6.552203$ |
| 2 | $s^2+2.372356s+3.314037$ |
| 3 | $(s^2+0.969406s+1.689747)(s+0.969406)$ |
| 4 | $(s^2+0.528313s+1.330031)(s^2+1.275460s+0.622925)$ |
| 5 | $(s^2+0.333067s+1.194937)(s^2+0.871982s+0.635920)(s+0.538914)$ |
| 6 | $(s^2+0.229387s+1.129387)(s^2+0.626696s+0.696374)(s^2+0.856083s+0.263361)$ |
| 7 | $(s^2+0.167682s+1.092446)(s^2+0.469834s+0.753222)(s^2+0.678930s+0.330217)(s+0.376778)$ |
| 8 | $(s^2+0.127960s+1.069492)(s^2+0.364400s+0.798894)(s^2+0.545363s+0.416210)$<br>$(s^2+0.643300s+0.145612)$ |
| 9 | $(s^2+0.100876s+1.054214)(s^2+0.290461s+0.834368)(s^2+0.445012s+0.497544)$<br>$(s^2+0.545888s+0.201345)(s+0.290461)$ |
| 10 | $(s^2+0.0815773s+1.043513)(s^2+0.236747+0.861878)(s^2+0.368742s+0.567985)$<br>$(s^2+0.464642s+0.274093)(s^2+0.515059s+0.0924569)$ |

Table A.5. **Denominator polynomials in factored form for Chebyshev filters**

$\alpha_p = 0.5$ dB $\quad (\epsilon^2 = 0.122018 \quad \epsilon = 0.349311)$

| $n$ | Polynomials |
|---|---|
| 1 | $s+2.862775$ |
| 2 | $s^2+1.425625s+1.516203$ |
| 3 | $(s^2+0.626456s+1.142448)(s+0.626456)$ |
| 4 | $(s^2+0.350706s+1.063519)(s^2+0.846680s+0.356412)$ |
| 5 | $(s^2+0.223926s+1.035784)(s^2+0.586245s+0.476767)(s+0.362320)$ |
| 6 | $(s^2+0.155300s+1.023023)(s^2+0.424288s+0.590010)(s^2+0.579588s+0.156997)$ |
| 7 | $(s^2+0.114006s+1.016108)(s^2+0.319439s+0.676884)(s^2+0.461602s+0.253878)(s+0.256170)$ |
| 8 | $(s^2+0.0872402s+1.011932)(s^2+0.248439s+0.741334)(s^2+0.371815s+0.358650)$<br>$(s^2+0.438586s+0.0880523)$ |
| 9 | $(s^2+0.0689054s+1.009211)(s^2+0.198405s+0.789365)(s^2+0.303975s+0.452541)$<br>$(s^2+0.372880s+0.156342)(s+0.198405)$ |
| 10 | $(s^2+0.0557988s+1.007335)(s^2+0.161934s+0.825700)(s^2+0.252219s+0.531807)$<br>$(s^2+0.317814s+0.237915)(s^2+0.352300s+0.0562789)$ |

Table A.7. Denominator polynomials in factored form for Chebyshev filters

$\alpha_p = 1$ dB $(\epsilon^2 = 0.258925 \quad \epsilon = 0.508847)$

| $n$ | Polynomials |
|---|---|
| 1 | $s + 1.965227$ |
| 2 | $s^2 + 1.097734s + 1.102510$ |
| 3 | $(s^2 + 0.494171s + 0.994205)(s + 0.494171)$ |
| 4 | $(s^2 + 0.279072s + 0.986505)(s^2 + 0.673739s + 0.279398)$ |
| 5 | $(s^2 + 0.178917s + 0.988315)(s^2 + 0.468410s + 0.429298)(s + 0.289493)$ |
| 6 | $(s^2 + 0.124362s + 0.990732)(s^2 + 0.339763s + 0.557720)(s^2 + 0.464125s + 0.124707)$ |
| 7 | $(s^2 + 0.0914180s + 0.992679)(s^2 + 0.256147s + 0.653456)(s^2 + 0.370144s + 0.230450)(s + 0.205414)$ |
| 8 | $(s^2 + 0.0700165s + 0.994141)(s^2 + 0.199390s + 0.723543)(s^2 + 0.298408s + 0.340859)$ $(s^2 + 0.351997s + 0.0702612)$ |
| 9 | $(s^2 + 0.0553349s + 0.995233)(s^2 + 0.159330s + 0.775386)(s^2 + 0.244108s + 0.438562)$ $(s^2 + 0.299443s + 0.142364)(s + 0.159330)$ |
| 10 | $(s^2 + 0.0448289s + 0.996058)(s^2 + 0.130099s + 0.814423)(s^2 + 0.202633s + 0.520530)$ $(s^2 + 0.255333s + 0.226637)(s^2 + 0.283039s + 0.0450019)$ |

Table A.9. Denominator polynomials in factored form for Chebyshev filters

$\alpha_p = 2$ dB $(\epsilon^2 = 0.584893 \quad \epsilon = 0.764783)$

| $n$ | Polynomials |
|---|---|
| 1 | $s + 1.307560$ |
| 2 | $s^2 + 0.803816s + 0.823060$ |
| 3 | $(s^2 + 0.368911s + 0.886095)(s + 0.368911)$ |
| 4 | $(s^2 + 0.209775s + 0.928675)(s^2 + 0.506440s + 0.221568)$ |
| 5 | $(s^2 + 0.134922s + 0.952167)(s^2 + 0.353230s + 0.393150)(s + 0.218308)$ |
| 6 | $(s^2 + 0.0939464s + 0.965952)(s^2 + 0.256666s + 0.532939)(s^2 + 0.350613s + 0.0999261)$ |
| 7 | $(s^2 + 0.0691327s + 0.974615)(s^2 + 0.193706s + 0.635391)(s^2 + 0.279913s + 0.212386)(s + 0.155340)$ |
| 8 | $(s^2 + 0.0529848s + 0.980380)(s^2 + 0.150888s + 0.709782)(s^2 + 0.225820s + 0.327099)$ $(s^2 + 0.266372s + 0.0565006)$ |
| 9 | $(s^2 + 0.0418943s + 0.984398)(s^2 + 0.120630s + 0.764552)(s^2 + 0.184816s + 0.427727)$ $(s^2 + 0.226710s + 0.131529)(s + 0.120630)$ |
| 10 | $(s^2 + 0.0339516s + 0.987304)(s^2 + 0.0985315s + 0.805669)(s^2 + 0.153466s + 0.511776)$ $(s^2 + 0.193379s + 0.217883)(s^2 + 0.214363s + 0.0362477)$ |

Table A.11. Denominator polynomials in factored form for Chebyshev filters

$\alpha_p = 3$ dB $(\epsilon^2 = 0.995262 \quad \epsilon = 0.997628)$

| $n$ | Polynomials |
|---|---|
| 1 | $s + 1.002377$ |
| 2 | $s^2 + 0.644900s + 0.707948$ |
| 3 | $(s^2 + 0.298620s + 0.839174)(s + 0.298620)$ |
| 4 | $(s^2 + 0.170341s + 0.903087)(s^2 + 0.411239s + 0.195980)$ |
| 5 | $(s^2 + 0.109720s + 0.936025)(s^2 + 0.287250s + 0.377009)(s + 0.177530)$ |
| 6 | $(s^2 + 0.0764590s + 0.954830)(s^2 + 0.208890s + 0.521818)(s^2 + 0.285349s + 0.0888048)$ |
| 7 | $(s^2 + 0.0562913s + 0.966483)(s^2 + 0.157725s + 0.627259)(s^2 + 0.227919s + 0.204254)(s + 0.126485)$ |
| 8 | $(s^2 + 0.0431563s + 0.974173)(s^2 + 0.122899s + 0.703575)(s^2 + 0.183931s + 0.320892)$ $(s^2 + 0.216961s + 0.0502939)$ |
| 9 | $(s^2 + 0.0341304s + 0.979504)(s^2 + 0.0982746s + 0.759658)(s^2 + 0.150565s + 0.422834)$ $(s^2 + 0.184696s + 0.126636)(s + 0.0982746)$ |
| 10 | $(s^2 + 0.0276639s + 0.983346)(s^2 + 0.0802838s + 0.801711)(s^2 + 0.125045s + 0.507818)$ $(s^2 + 0.157566s + 0.213926)(s^2 + 0.174663s + 0.0322899)$ |